COMP 233 Probability and Statistics for Computer Science

Fall 2016, Assignment 2 Due: October 14, 2016

Question 1

A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.

Question 2

Two ballpoint pens are selected at random from a box that contains 3 blue pens, 2 red pens, and 3 green pens. If X is the number of blue pens selected and Y is the number of red pens selected.

(a) Find the joint probability function f(x, y),
(b) Find E[X], E[Y], Var(X), and Var(Y).
(c) Find Cov(X,Y), Var(X + Y), and Corr(X,Y).
(d) P[(X, Y)∈A], where A is the region $\{(x, y) \mid x+y \leq 1\}$.

Question 3

If X is a continuous random variable having cumulative distribution F, then its median is defined as that value of m for which

$$F(m) = 1/2.$$

Consider the random variable X with density function

$$f(x) = 3x^2, 0 \leq x \leq 1.$$

a) Find the median of X.
b) Find the expectation of X.

Question 4

An electrical firm manufactures 100-watt light bulbs. These bulbs, according to specifications written on the package, have a mean life of 900 hours with a standard deviation of 50 hours. Assume that the distribution is symmetric about the mean.

a) Give an upper bound to the probability that the life of a bulb win a new production batch will exceed 990 hours.
b) Suppose that the mean is unknown but the standard deviation is 50 hours. What can be said about the probability that the life of a new bulb will be within 150 hours of the unknown mean for that batch?
c) How many new bulbs should be made so as to ensure, with probability at least 0.75, that the average life would be within 25 hours of the unknown mean? Assume each bulb is produced independently.

Question 5

The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive, (b) from 3 to 8 survive, and (c) exactly 5 survive? (Use a Cumulative binomial probabilities table.)

Question 6

During a laboratory experiment, the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is the probability that 6 particles enter the counter in a given millisecond? (Use a Cumulative binomial probabilities table.)

Question 7

The average grade for an exam is 74, and the standard deviation is 7. If 12% of the class is given As, and the grades are curved to follow a normal distribution, what is the lowest possible A and the highest possible B? (Use a Cumulative binomial probabilities table.)

Question 8

The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. What is the probability that a person is served in less than 3 minutes on at least 4 of the next 6 days?