Université d'Ottawa
Faculté de génie
Département de
génie mécanique

uOttawa

University of Ottawa
Faculty of Engineering
Department of
Mechanical Engineering

MCG 3305: Biomedical Systems Dynamics
Midterm Exam

Prof. Davide Spinello

Wednesday, October 12, 2016 **Duration: 1hr20m**

Policy

The present exam is open book and open notes. Illegible work and loose sheets will not be graded. All electronic devices, with the exception of non programmable calculators, must be turned off during the test.

Problem 1

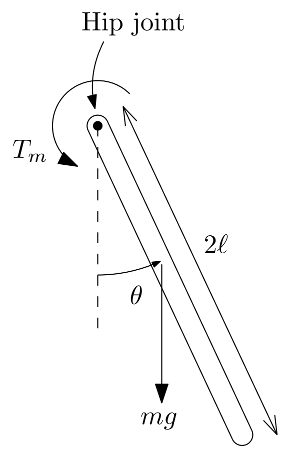

Fig. 1: Mechanical model of a human leg.

A simplified model of human leg is sketched in Fig. 1. The model assumes an applied muscular torque, $T_m(t)$, viscous damping, b, at the hip joint, and inertia, J, around the hip joint. Note that the moment of inertia is J is around the hip joint, NOT around the centre of mass: this affects the way you write Newton's second law.

1. (**10pt**) Derive the mathematical model of the system.
2. (**5pt**) Linearize the model.
3. (**10pt**) Obtain the transfer function $\Theta(s)/T_m(s)$.
4. (**5pt**) Obtain a state space representation in which T_m is the input.

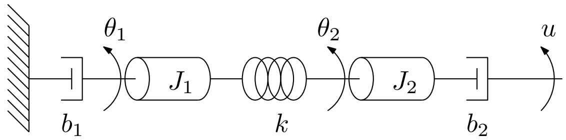

Fig. 2: Rotational mechanical system.

Problem 2

Consider the two degrees of freedom rotational system in Figure 2, where the input u is a constant angular velocity, and the torque at the extreme of rotational dampers is proportional to the difference in angular velocity of the extremes (fully analogous to linear dampers). θ_1 and θ_2 are angular positions of the two rotating masses, b_1 and b_2 are the damping coefficients of the rotational dampers, and k is the stiffness of the rotational spring.

1. (**10pt**) Obtain the mathematical model of the system in terms of ODEs. (*Hint:* you can consider the analogy with traslational mechanical systems as linear displacement-rotation, linear velocity $\dot{x}$-angular velocity $\dot{\theta}$, linear spring-rotational spring, linear damper-rotational damper, mass-moment of inertia.)
2. (**10pt**) Obtain a state space representation of the system.

Problem 3

HIV operates by infecting healthy CD4+T cells (a type of white blood cells) that are necessary to fight the infection. The virus embeds in a T cell, which results into production of more T cells to fight the infection, which in turn allows the virus to spread opportunistically. A simple mathematical model describing this is (*Craig, 2004*):

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \tau - \delta T - \beta T v \tag{1}$$

$$\frac{\mathrm{d}T^\star}{\mathrm{d}t} = \beta T v - \mu T^\star \tag{2}$$

$$\frac{\mathrm{d}v}{\mathrm{d}t} = kT^\star - cv \tag{3}$$

where constants τ and δ are production and death rates of T cells, constant β is the rate of infection, constant k is the rate of reproduction and constant c is the rate of death of free viruses, whereas constant μ is the rate of death of infected T cells. The variables of the system are T (number of healthy T cells), $T^\star$ (number of infected T cells), and v (number of free viruses).

1. (**10pt**) Determine which equations are linear, which are nonlinear, and explain why.
2. (**5pt**) Linearize the system around $T = T^\star = v = 0$.
3. (**5pt**) The equilibrium is obtained by setting the time derivatives to zero. Calculate the equilibrium of the linearized system by solving for T, $T^\star$, and v.