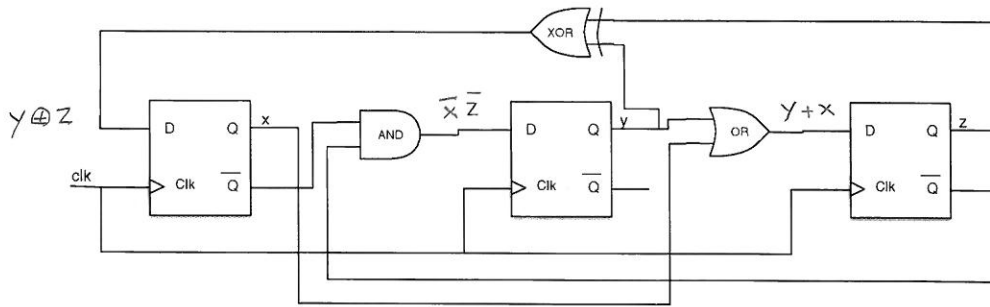
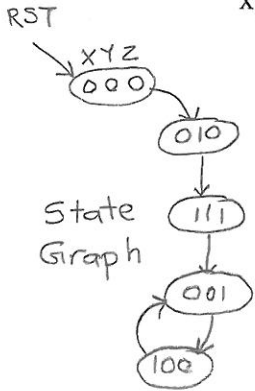
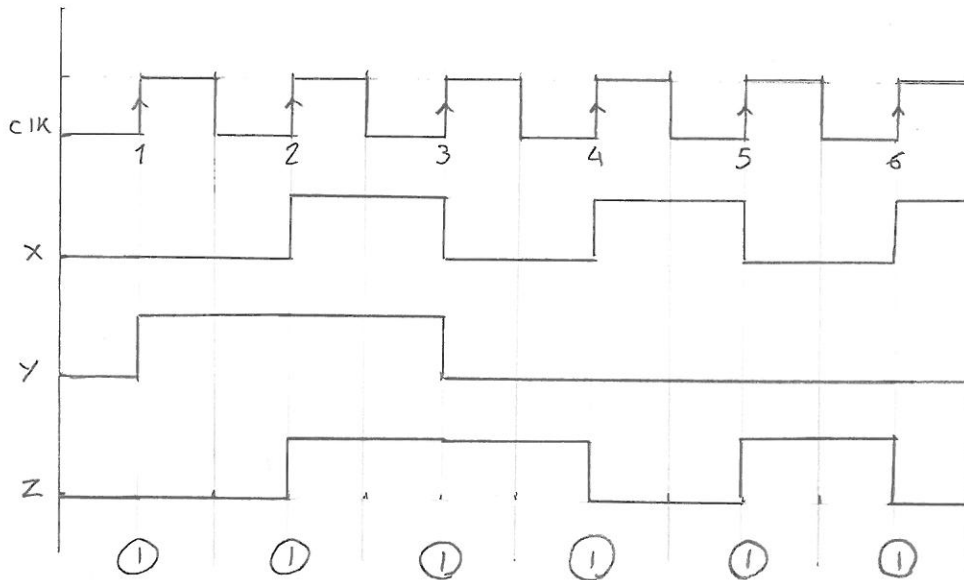


1. Plot the timing diagram for the following circuit for 6 clk cycles. Assume that initially $xyz=000$. Plot the clk, x, y, and z signals. 6 marks

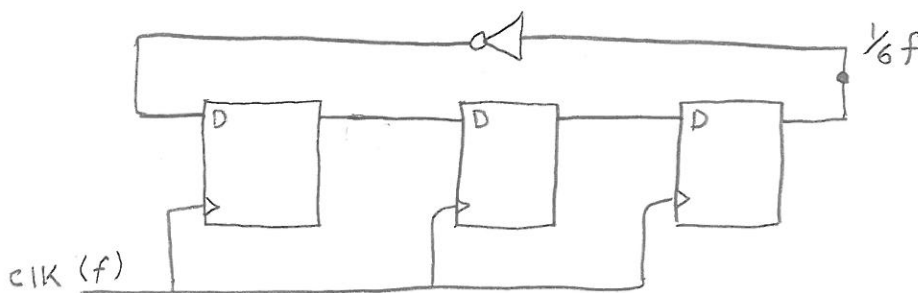


$$\begin{aligned}
 x^+ &= y \oplus z \\
 y^+ &= \bar{x} \bar{z} \\
 z^+ &= y + x
 \end{aligned}$$



clk edge	x	y	z	
	0	0	0	→ start
1	0	1	0	
2	1	1	1	
3	0	0	1	
4	1	0	0	
5	0	0	1	
6	1	0	0	

2. Design a circuit made of rising-edge-triggered flip-flops and inverter(s) such that its output has one-sixth ($1/6$) of the clock frequency. Verify your design by a timing diagram. The timing diagram should include the clk signal and the outputs of the flip-flops and logic gates. 3 marks



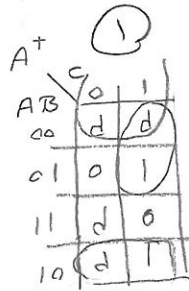
3. Design a circuit that outputs prime numbers between 0 and 10 in ascending order and repeats. Try to use the minimum number of flip-flops possible in your design. Show your state-graph, state-table, next-state and output equations, and the final circuit. 7 marks

prime numbers : 2, 3, 5, 7

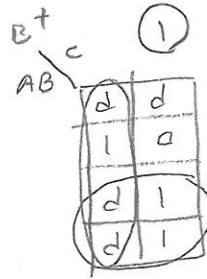
⇒ we need 4 states

①

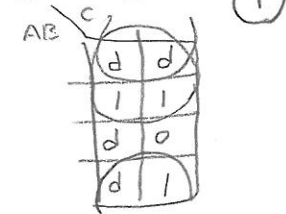
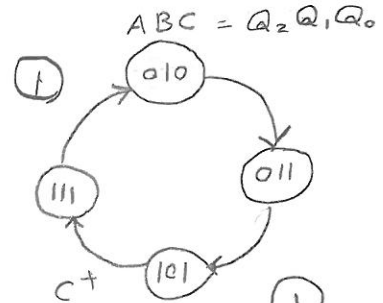
state ABC	next state A ⁺ B ⁺ C ⁺
000	d d d
001	d d d
011	1 0 1
010	0 1 1
100	d d d
101	1 1 1
111	0 1 0
110	d d d



$$A^+ = \bar{B} + \bar{C} + AC$$



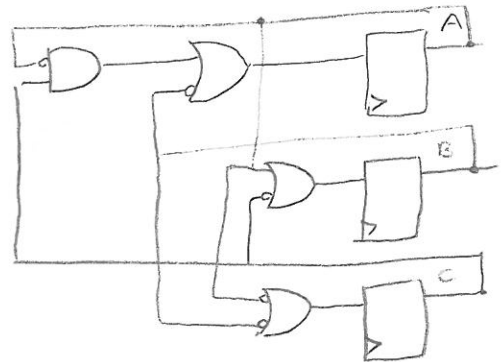
$$B^+ = \bar{C} + A$$



$$C^+ = \bar{A} + \bar{B}$$

①

uses 3 FlipFlops + 4 logic gates



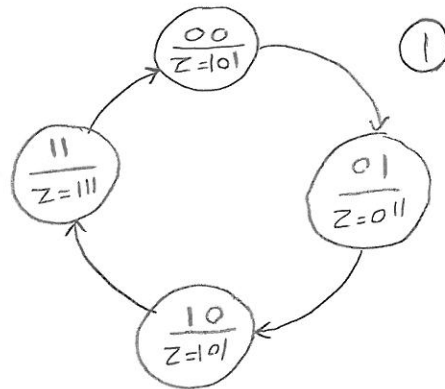
①

Q3. Alternative Solution

In the previous solution the state variables of the FSM were the outputs of the counter as well. Here, we assume that the outputs are produced by the state variables.

①

state Q_1, Q_0	Next state Q_1^+, Q_0^+	output Z_2, Z_1, Z_0
00	01	010
01	10	011
10	11	101
11	00	111



① $Q_0^+ = \bar{Q}_0$

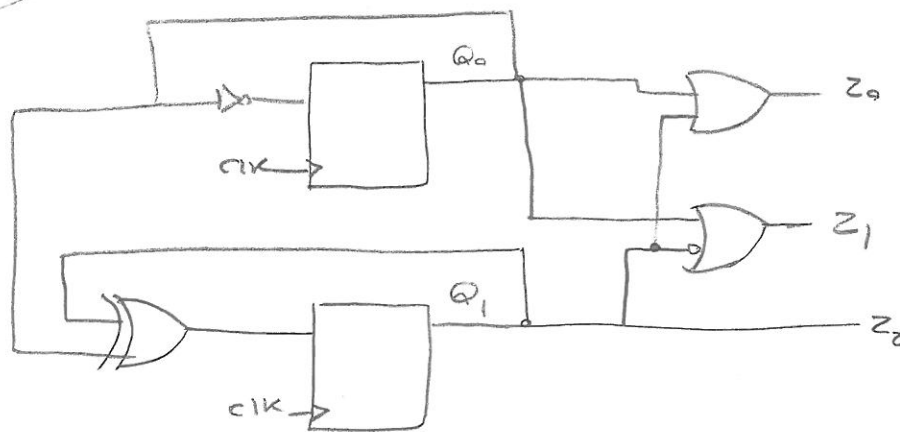
$Q_1^+ = Q_1 \oplus Q_0$

① $Z_0 = Q_1 + Q_0$

① $Z_1 = \bar{Q}_1 + Q_0$

① $Z_2 = Q_1$

①



uses 2 Flipflops + 3 Logic gates

This is more efficient.

4. Design and draw the state-graph of an FSM that detects the input sequence '11011' including overlaps. The sequence appears at the input from left to right. Suggest two different designs and compare them by listing the advantage and disadvantage of each. For both designs provide a state diagram, state-table, K-Maps, equations, and the circuit. 14 marks

See the solution on next pages

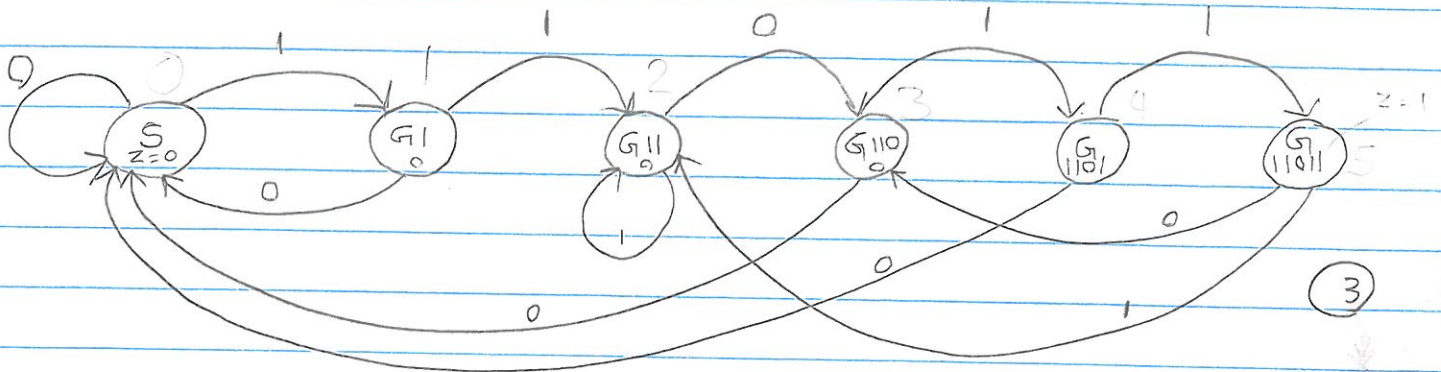
- Comparison

criteria	Moore	Mealy
# of gates	12	9
# of F/F	3	3
output glitch	no	Yes
short pulses	no	Yes
response	slower	faster

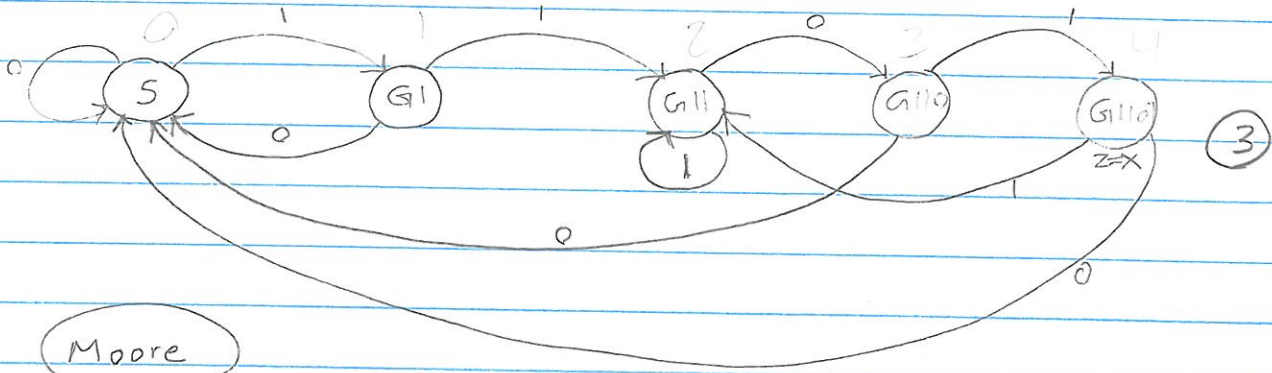
Mealy circuit design is more efficient/faster but less reliable. (2)

Q4.

Moore FSM Graph ↓



Mealy FSM Graph ↓



Moore

$Q_2 Q_1 Q_0$	$x=0$		$x=1$		Z	D_0				D_1				
	$D_2 D_1 D_0$	$D_2 D_1 D_0$	$D_2 D_1 D_0$	$D_2 D_1 D_0$		$Q_2 Q_1 Q_0$	$Q_2 x$	01	11	10	00	01	11	10
000	000	000	001	000	0	00	0	1	1	0	00	0	0	0
001	000	000	010	000	0	01	0	0	0	1	01	0	1	1
011	000	000	100	000	0	11	0	0	d	d	11	0	d	d
010	011	011	010	010	0	10	1	0	d	d	10	1	1	d
100	000	000	101	101	0									
101	011	011	010	010	1									
111	d d d	d d d	d d d	d d d	d									
110	d d d	d d d	d d d	d d d	d									

$$D_0 = \bar{Q}_1 \bar{Q}_0 x + Q_2 \bar{x} Q_0 + Q_1 Q_0 \bar{x}$$

$$D_1 = \bar{Q}_1 Q_0 x + Q_0 Q_2 + Q_1 Q_0$$

D_2	Q_2	Q_1	Q_0	x
00	0	0	1	0
01	0	0	0	0
11	0	1	1	d
10	0	0	1	d

$$D_2 = Q_1 Q_0 x + \bar{Q}_0 Q_2 x$$

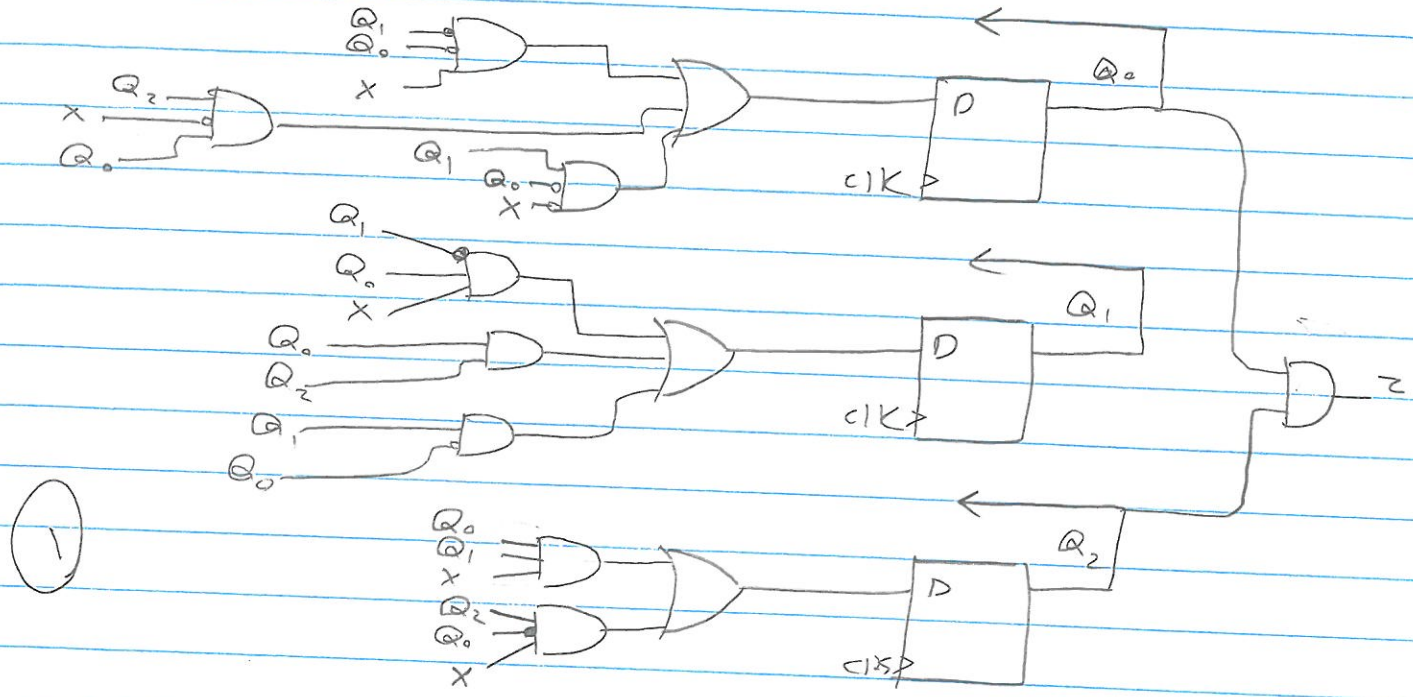
Z	Q_2	Q_0
00	0	0
01	0	1
10	0	d
11	0	d

$$Z = Q_2 Q_0$$

2

1/2 for each k-map

Moore Circuit



(1)

Mealy

State $Q_2 Q_1 Q_0$	next state			output	
	$X=0$ $D_2 D_1 D_0$	$X=1$ $D_2 D_1 D_0$	$X=0$ Z	$X=1$ Z	
0 0 0	0 0 0	0 0 1	0	0	
0 0 1	0 0 0	0 1 0	0	0	
0 1 1	0 0 0	1 0 0	0	0	
0 1 0	0 1 1	0 1 0	0	0	
1 0 0	0 0 0	0 1 0	0	1	
1 0 1	d d d	d d d	d	d	
1 1 1	d d d	d d d	d	d	
1 1 0	d d d	d d d	d	d	

$Q_2 Q_1$	$Q_2 X$	00	01	11	10
00	0	0	0	1	0
01	0	0	0	d	d
11	0	0	0	d	d
10	0	0	0	d	d

$Z = Q_2 X$ (1/5)

$Q_2 Q_1$	$Q_2 X$	00	01	11	10
00	0	0	1	0	0
01	0	0	0	d	d
11	0	0	0	d	d
10	1	0	0	d	d

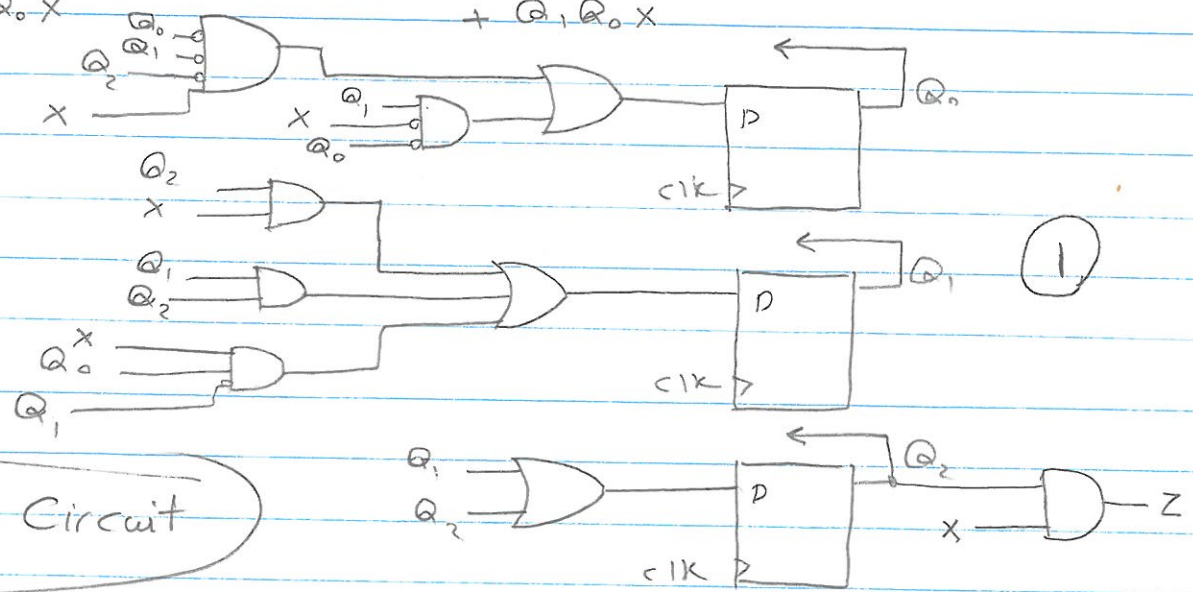
$Q_2 Q_1$	$Q_2 X$	00	01	11	10
00	0	0	0	1	0
01	0	0	1	d	d
11	0	0	0	d	d
10	1	1	d	d	d

$Q_2 Q_1$	$Q_2 X$	00	01	11	10
00	0	0	0	0	0
01	0	0	0	d	d
11	0	0	1	d	d
10	0	0	0	d	d

$D_0 = \bar{Q}_2 \bar{Q}_1 \bar{Q}_0 X + \bar{Q}_2 \bar{Q}_1 X$

$D_1 = Q_2 X + \bar{Q}_2 Q_0$

$D_2 = Q_1 Q_2$



Mealy Circuit