Solution

ELEC 2607 **Assignment 1** 20 Marks

1. Using the method of exhaustive proof show whether $\overline{a+b+c}$ is equal to $\overline{(a+b)} \cdot \bar{c}$.

2 marks

$\overline{a+b+c}$:

c \ ab	00	01	11	10
0	1	0	0	0
1	0	0	0	0

(1)

$\overline{(a+b)} \cdot \bar{c}$:

c \ ab	00	01	11	10
0	1	0	0	0
1	0	0	0	0

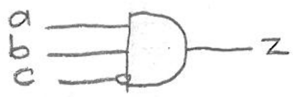
(1)

Same compact truth table ⇒ equal

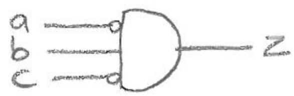
2. Draw the simplest logic gate or circuit for the following descriptions. The inputs are a, b, c. The output is z.

4 marks

(1) (a) $z = 1$ only if $abc = 110$

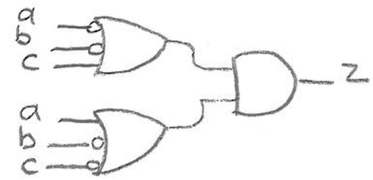


(1) (b) $z = 0$ unless $abc = 010$



(2) (c) $z = 0$ only if "$abc = 110$ or $abc = 011$"

$\bar{z} = ab\bar{c} + \bar{a}bc \Rightarrow z = (\bar{a}+\bar{b}+c)(a+\bar{b}+\bar{c})$



3. Simplify the following

4 marks

(1) (a) $(X + Y + \bar{Y}Z)(Y + X)(Y + \bar{X})$ to 1 literal

$= (X+Y+\bar{Y}Z)(YY+Y\bar{X}+XY+X\bar{X}) = (X+Y+\bar{Y}Z)(Y+Y\bar{X}+YX+0)$

$= (X+Y+\bar{Y}Z)[Y+Y(X+\bar{X})+0] = (X+Y+\bar{Y}Z)Y = Y$

(1) (b) $(BCD + C)CD$ to 2 literals

$= BCDCD + CCD = BCD + CD = CD(B+1) = CD$

(2) (c) $A\bar{B} + AC + \bar{C}B$ to 3 literals

according to consensus: $AC + \bar{C}B = AC + AB + \bar{C}B$

$\Rightarrow A\bar{B} + AC + \bar{C}B = A\bar{B} + AB + AC + \bar{C}B = A + AC + \bar{C}B$

$= A + \bar{C}B$

one may also simplify by expressing as $A\bar{B}(C+\bar{C}) + AC + \bar{C}B$

4. Prove algebraically that $(a + b)(b + c)(c + a) = ab + bc + ca$

2 mark

$LHS = (ab + ac + bb + bc)(c+a) = (ab + ac + b + bc)(c+a)$

$= [b(a+1+c) + ac](c+a) = (b+ac)(c+a)$

$= bc + ab + acc + aac = bc + ab + ca = RHS$

5. Use duality on $\bar{a}\bar{b} + ab = (a + \bar{b})(\bar{a} + b)$ to find an alternate expression for $\bar{a}b + a\bar{b}$.

1 mark

$(\bar{a}+\bar{b})(a+b) = a\bar{b} + \bar{a}b$ by applying duality on both sides.

$\Rightarrow (\bar{a}+\bar{b})(a+b)$ is another expression for $a\bar{b}+\bar{a}b = a \oplus b$

6. Convert the following expressions to a form where the bar is over single variables only.

4 marks

(2) (a) $\overline{a(b\bar{c} + de) + \overline{(d+a)}cg} = \overline{a(b\bar{c}+de) + (\bar{d}\bar{a}\cdot cg)}$

$= \overline{\{a(\{b\bar{c}\} + \{de\})\} + \{\bar{a}\bar{d}cg\}}$

$= \{\bar{a} + (\{\bar{b}+c\}\cdot\{\bar{d}+\bar{e}\})\} + \{a+d+\bar{c}+\bar{g}\} = [\bar{a} + (\bar{b}+c)(\bar{d}+\bar{e})] + (a+d+\bar{c}+\bar{g})$

(2) (b) $\overline{a(b\bar{c} + \overline{e}d) + \overline{\overline{\overline{(d+ab)}\;\overline{(cg)}}}} + a\overline{dcg}$

$= \overline{a[b\bar{c} + (e+\bar{d})] + [(d+ab)+(cg)] + [a(\bar{d}+\bar{c}+\bar{g})]}$

$= \overline{\{a[(b\bar{c})+e+\bar{d}]\} + [d+(ab)+(cg)] + [a(\bar{d}+\bar{c}+\bar{g})]}$

$= \{\bar{a} + [(\bar{b}+c)\cdot\bar{e}\cdot d]\}\cdot[\bar{d}\cdot(\bar{a}+\bar{b})\cdot(\bar{c}+\bar{g})]\cdot[\bar{a}+(dcg)]$

7. Derive the equations for y and z from the following table and simplify the answer as much as possible. Then implement y and z using a minimum number of 2-input and 3-input gates (note: inverters (bubbles) don't count). Also identify the function obtained by y and z (eg. is the function an 'and', 'xor', 'or', 'majority gate' etc.?)

3 marks

c	b	a	y	z
0	0	0	1	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	0

$Y = \bar{a}\bar{b}\bar{c} + ab\bar{c} + a\bar{b}c + \bar{a}bc$

$= \bar{c}(\bar{a}\bar{b} + ab) + c(a\bar{b} + \bar{a}b)$

$= \bar{c}\,\overline{(a\oplus b)} + c(a\oplus b)$

$= \overline{c\oplus(a\oplus b)}$

$= \overline{a\oplus b\oplus c}$ XNOR

$Z = \bar{a}\bar{b}\bar{c} + a\bar{b}\bar{c} + \bar{a}b\bar{c} + \bar{a}\bar{b}c$

$= \bar{a}\bar{b}(c+\bar{c}) + a\bar{b}\bar{c} + \bar{a}b\bar{c}$

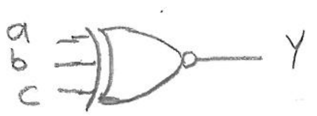
$= \bar{a}\bar{b} + a\bar{b}\bar{c} + \bar{a}b\bar{c}$

$= \bar{a}\bar{b} + a\bar{b}\bar{c} + \bar{a}b\bar{c} + \bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}\bar{c}$

$= \bar{a}\bar{b} + \bar{b}\bar{c} + \bar{a}\bar{c}$

$= \overline{ab + bc + ac}$

$= \overline{\text{Majority}(a, b, c)}$



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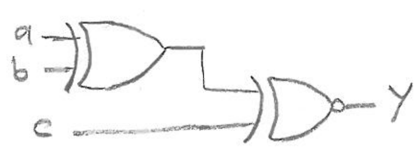

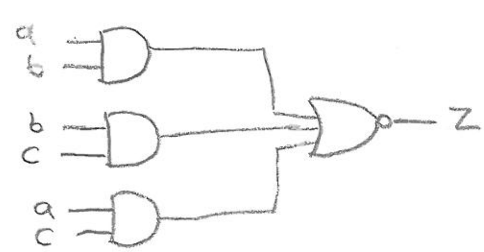

note