Concordia University
ENCS
Engineering & Computer Science

FACULTY OF ENGINEERING AND COMPUTER SCIENCE

COURSE		NUMBER	SECTION
Numerical Methods in Engineering		**ENGR 391**	**/4 (all)**
EXAMINATION	DATE	TIME	# of pages (including title page)
Final Exam	**Dec. 9, 2013**	**14 :00-17 :00**	**18**
PROFESSORS			
Dr.D. Davis, Dr. P. Gauthier and Dr. A. Kaushal			
MATERIALS ALLOWED - YES (One page **single-sided** crib sheet) CALCULATORS ALLOWED - YES (Faculty approved calculators)			
SPECIAL INSTRUCTIONS: • Read carefully all the questions • Total Marks – 100; Time 180 Minutes • Closed Book Exam; **Single sided** 8 ½ x 11 formula sheet allowed • You must **show all your steps** leading to the solution(s) • Give the answers in the area provided. • Please do not write in red (colour used for correction) • Everything not readable will NOT be corrected **Good Luck!**			

Name: _____ **I.D.:** _____
Surname, Given names

Signature: _____

MARKS

Question #1 [Solving Nonlinear Equations] [10 marks]

Obtain the first root above x = 0 for the following equation with accuracy of 4 digits
(Hint: use incremental search to locate the region of the root)

$$e^x - 2x^2 = 0$$

a. Use the method of False Position (5 Marks)
b. Use Newton Raphson method (5 Marks)

Question #2 [Systems of linear and Nonlinear equations] [25 marks]

a) Consider the following system of linear equations $[A]\{X\} = \{B\}$

$$\begin{bmatrix} 1 & 4 & 2 \\ 5 & 2 & 1 \\ 2 & 2 & 8 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 1 \\ -2 \\ 2 \end{Bmatrix}$$

If using decimals instead of fraction number, keep **3 decimals** in your calculations

1. Find the solution of this system of linear equations **using the LU decomposition** with **partial pivoting** (i.e. PA = LU). (10 Marks)
2. Find the first column of $[A]^{-1}$ (5 Marks)

b) Obtain the solution to the following nonlinear equations using Newton's Method of the form $\{X_{n+1}\} = \{X_n\} - [J(X_n)]^{-1}\{F(X_n)\}$

$$-x_1 + 2x_1^2 - 2x_1x_2 + x_2^2 = 1$$

$$x_1^2 - 2x_1x_2 - x_2 + x_2^2 = 0$$

Assume the starting vector $\{x_0\} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$. Do **<u>two iterations</u>**. Compute the error **<u>at each step</u>** of the iteration using the $\|x\|_2$ norm; Use **4 decimals** in your calculations (10 Marks)

Question #3 [Curve Fitting] [20 marks]

a) If the following points are related by a formula of the type , $P(x) = Ae^{Mx}$
Find the best value of A and M; Keep **4 decimals** in your calculations. (10 Marks)
(Hint: Change the form to a linear equation and use least squares regression)

x_i	1	2	3	4
P_i	7	11	17	27

b) Use the Lagrange Interpolating Polynomial to approximate Cos (0.750) using the following values; (Note the given values are in radians) (10 Marks)

Cos(0.698) = 0.7661 Cos(0.733) = 0.7432 Cos(0.768) = 0.7193

Question #4 [Numerical Differentiation & Integration] [25 marks]

a) Evaluate the Integral:

$$I = \int_0^{2.4} \frac{2x}{x^2+1}\,dx$$

Gauss Quadrature

$n = 4$:

$c_1 = 0.3478548; \quad x_1 = -0.86113631;$

$c_2 = 0.6521452; \quad x_2 = -0.33998104;$

$c_3 = c_2; \quad x_3 = -x_2;$

$c_4 = c_1; \quad x_4 = -x_1$

1. Analytically (2.5 Marks)
2. Using Simpson's 1/3 method, using 6 sub-intervals. (5 Marks)
3. Using four-point Gauss Quadrature (5 Marks)
4. Using the exact solution found in part a) evaluate the percent relative error associated with each of the approximations found in parts 2) and 3) (2.5 Marks)

Keep **3 decimals** in your calculations.

b. The following data is given for the stopping distance of a truck on a wet road versus the speed at which it begins breaking:

v (km/h)	20.0	40.5	62.5	80	100.5	125
d(m)	6	19	38	65	99	135

1. Calculate the rate of change of the stopping distance at a speed of 100.5 km/h using a two-point backward difference formula. (5 Marks)
2. Estimate the stopping distance at 125 km/h using the result from part 1) and a two-point central difference formula applied at the speed of 100.5 km/h. (5 Marks)

Question #5 [Ordinary Differential Equations] [20 marks]

a. Given the differential equation

$$\frac{dy}{dx} = \frac{4x}{y} - xy$$

Fill out the following table using the classical fourth- order Runge Kutta method; (10 Marks)
Keep **6 decimals** in your calculations.

Xi	Yi
0.00000	3.000000
0.10000	
0.20000	

b) Solve numerically using Euler's method the following second order ordinary differential equation with a step size, *h = 0.5* , from *t=1 to t=2 ,for given* initial conditions:

(10 Marks)

$$\frac{d^2y}{dt^2} = 4y^3$$

y(1)=1/4 and *dy/dt(1)=0*

Keep **5 decimal** places in your calculations

EXTRA SHEETS