

Question 1- Calculate the following one-sided limit:

$$\lim_{x \rightarrow -1^+} \frac{x}{x^2 - 1} = \frac{x}{(x+1)(x-1)} \quad \frac{-}{+ -} = +$$

- A) 0 B) 1 C) ∞ D) $-\infty$ E) -1

Sample point
= -0.99

Question 2- Suppose that the variable y is implicitly defined as a function of x by the equation $(x^2 - y)^2 = 2y$. Find $\frac{dy}{dx}$ at the point $(2, 2)$.

- A) $\frac{1}{2}$ B) 3 C) -1 D) $\frac{8}{3}$ E) $\frac{5}{2}$

$$(x^2 - y)^2 = 2y$$

$$2(x^2 - y)(2x - \frac{dy}{dx}) = 2 \frac{dy}{dx}$$

Plug in (2, 2)

$$2(2)(4 - \frac{dy}{dx}) = 2 \frac{dy}{dx}$$

$$16 - 4 \frac{dy}{dx} = 2 \frac{dy}{dx}$$

$$16 = 6 \frac{dy}{dx}$$

$$\frac{8}{3} = \frac{dy}{dx}$$

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Question 3- Let $h(x) = x \ln(x)$. Find the equation of the tangent line at $x = e^2$.

- A) $y = ex - 1$ B) $y = 3x - 2e^2$ **C) $y = 3x - e^2$** D) $y = 3x - e$ E) $y = ex + 2$

$$h'(x) = \ln(x) + x \cdot \frac{1}{x} = \ln(x) + 1$$

$$h'(e^2) = \ln(e^2) + 1 = 2 + 1 = 3 \quad \leftarrow h'(e^2)$$

So $y = 3x + b$. Plug in $(e^2, 2e^2)$

$$2e^2 = 3e^2 + b$$

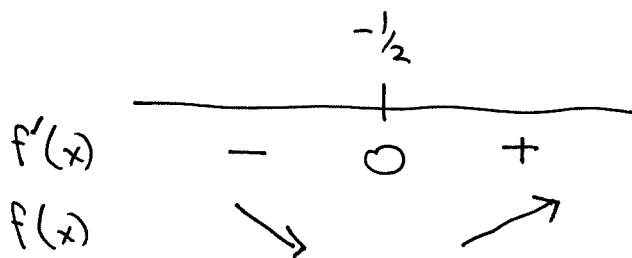
$$b = -e^2$$

Question 4- Over what interval is the function $f(x) = xe^{2x}$ decreasing?

- A) $(\frac{1}{2}, \infty)$ **B) $(-\infty, -\frac{1}{2})$** C) $(-1, 1)$ D) $(-1, \infty)$ E) $(0, 1)$

$$f'(x) = e^{2x} + 2xe^{2x} = (1+2x)e^{2x} \quad \text{So one CP}$$

$$\text{at } x = -\frac{1}{2}.$$



Question 5- Consider the function:

$$f(x) = \frac{5x + 2}{x - 3}$$

Which of the following statements are correct?

- A) The asymptotes of f occur at $y = 5, x = 3$.
- B) The asymptotes of f occur at $y = 3, x = 5$.
- C) $x = 3$ is the only asymptote of f .
- D) $y = 3$ is the only asymptote of f .
- E) The function has no asymptotes.

$\lim_{x \rightarrow \infty} f(x) = 5$. So a HA at $y = 5$
There is a VA at $x = 3$

Question 6- Evaluate:

$$\int_{\ln(2)}^{\infty} e^{-x} dx$$

- A) 1 **B) $\frac{1}{2}$** C) $\ln(2)$ D) $2\ln(2)$ E) divergent

$$\int e^{-x} dx = -e^{-x}. \text{ So } \int_{\ln(2)}^{\infty} e^{-x} dx = \lim_{b \rightarrow \infty} \int_{\ln(2)}^b e^{-x} dx$$

$$= \lim_{b \rightarrow \infty} -e^{-x} \Big|_{\ln(2)}^b = \lim_{b \rightarrow \infty} \left[-e^{-b} - (-e^{-\ln(2)}) \right]$$

$$= \lim_{b \rightarrow \infty} \left[\frac{1}{2} - \frac{1}{e^b} \right] = \frac{1}{2}$$

Question 7- Suppose $f'(x) = 3x^2 + 4x - 1$ and $f(1) = 3$. Find $f(2)$.

- A) 13 **B) 15** C) 19 D) 37 E) 51

$$f(x) = \int (3x^2 + 4x - 1) dx = x^3 + 2x^2 - x + C$$

$$f(1) = 2 + C = 3 \Rightarrow C = +1$$

$$\text{So } f(x) = x^3 + 2x^2 - x + 1$$

$$f(2) = 8 + 8 - 2 + 1 = 15$$

Question 8- Suppose that for a certain product, the demand function is given by $D(x) = 8 - x^2$ and the supply function is given by $S(x) = 2x$. Calculate the consumer surplus.

- A) 3 B) $\frac{11}{3}$ **C) $\frac{16}{3}$** D) 1 E) $\frac{25}{7}$

Equilibrium Point $D(x) = S(x)$

$$8 - x^2 = 2x \Rightarrow x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

$$\text{So EP is } (2, 4)$$

$$\text{So } CS = \int_0^2 (8 - x^2) - 4 dx = \int_0^2 (4 - x^2) dx = 4x - \frac{x^3}{3} \Big|_0^2$$

$$= 8 - \frac{8}{3} = \frac{16}{3}$$

Question 9- If $f(x, y) = y \ln(xy)$, what is $f_{xy}(2, 2)$?

- A) $\frac{4}{3}$ B) $\frac{-1}{4}$ C) $\frac{3}{2}$ **D) $\frac{1}{2}$** E) $\frac{e}{3}$

$$f_x = y \cdot \frac{1}{xy} \cdot (y) = \frac{y}{x}$$

$$f_{xy} = \frac{1}{x}$$

$$f_{xy}(2, 2) = \frac{1}{2}$$

Question 10- If $f(x, y) = x^3 - 4x + y^2 - 3y + 1$, how many critical points does $f(x, y)$ have?

- A) 0 B) 1 C) 2 D) 3 E) 4

$$f_x = 3x^2 - 4 = 0 \Rightarrow x^2 = \frac{4}{3} \Rightarrow x = \pm \sqrt{\frac{4}{3}}$$

$$f_y = 2y - 3 = 0 \Rightarrow y = \frac{3}{2}$$

So there are 2 CPs at $(\sqrt{\frac{4}{3}}, \frac{3}{2})$ & $(-\sqrt{\frac{4}{3}}, \frac{3}{2})$

Long Answer Question 1 (10 points)

Suppose that 10,000 dollars is put into a bank account which earns 4% interest compounded continuously.

- How much money will there be in the account after 6 years?
- How many years will it take until there is 40,000 dollars in the account?

$$\bullet \quad P(t) = P_0 e^{rt} = 10,000 e^{.04t}$$

$$P(6) = 10,000 e^{.24}$$

Do not simplify

$$\bullet \quad \text{Solve } 40,000 = 10,000 e^{.04t}$$

$$4 = e^{.04t}$$

$$\ln(4) = .04t$$

$$t = \frac{\ln(4)}{.04}$$

Long Answer Question 2 (12 points)

Calculate the following two indefinite integrals:

$$\int x e^{3x} dx$$

$u = x$	$v = \frac{1}{3} e^{3x}$
$du = dx$	$dv = e^{3x} dx$

$$\begin{aligned} \int x e^{3x} dx &= \frac{x}{3} e^{3x} - \int \frac{1}{3} e^{3x} dx \\ &= \frac{x}{3} e^{3x} - \frac{1}{9} e^{3x} + C \end{aligned}$$

$$\int \frac{\ln(x)}{x^2} dx$$

$u = \ln(x)$	$v = \frac{-1}{x}$
$du = \frac{1}{x} dx$	$dv = \frac{1}{x^2} dx$

$$\begin{aligned} \int \frac{\ln(x)}{x^2} dx &= \frac{-1}{x} \ln(x) - \int \left(\frac{-1}{x}\right) \left(\frac{1}{x}\right) dx = \\ &= \frac{-1}{x} \ln(x) + \int \frac{1}{x^2} dx = \frac{-1}{x} \ln(x) - \frac{1}{x} + C \end{aligned}$$

Long Answer Question 3 (12 points)

A store has been selling skateboards at \$40 per board. At this price they sell 50 boards per month. For each 1 dollar increase in price, they sell 2 fewer boards. If each board costs the store \$25, what price will maximize profit?

Be sure to explain why your answer is an absolute maximum.

Let $x = \#$ of skateboards / month
 $P = \text{Price} / \text{skateboard}$

P	x
40	50
41	48

$$m = \frac{\Delta P}{\Delta x} = \frac{-1}{2} \quad \text{So } P = -\frac{1}{2}x + b \quad \text{Plus in } (50, 40)$$

$$40 = -25 + b \Rightarrow b = 65$$

$$\text{So } P = -\frac{1}{2}x + 65$$

$$R(x) = xP = -\frac{1}{2}x^2 + 65x$$

$$C(x) = 25x$$

$$P(x) = R(x) - C(x) = -\frac{1}{2}x^2 + 40x$$

$$P'(x) = -x + 40. \quad \text{So one CP at } \cancel{x=40} = x=40$$

It is an absolute max, since $P(x)$ is a CD parabola.

$$\text{Corresponding price is } -\frac{1}{2}(40) + 65 = 45\$$$

Long Answer Question 4 (14 points)

Consider the two functions:

$$f(x) = 5 - x \quad \text{and} \quad g(x) = 3 + x^2$$

- (a) (4 points) Find the intersection points of the graphs of the two functions.
- (b) (6 points) On the next page, graph these functions, and shade the region between the graphs of f and g for $x \in [0, 4]$.
- (c) (6 points) Find the area of the shaded region.

$$a) \quad 5 - x = 3 + x^2 \Rightarrow x^2 + x - 2 = 0 \Rightarrow x = -2, 1$$
$$(x+2)(x-1) = 0$$

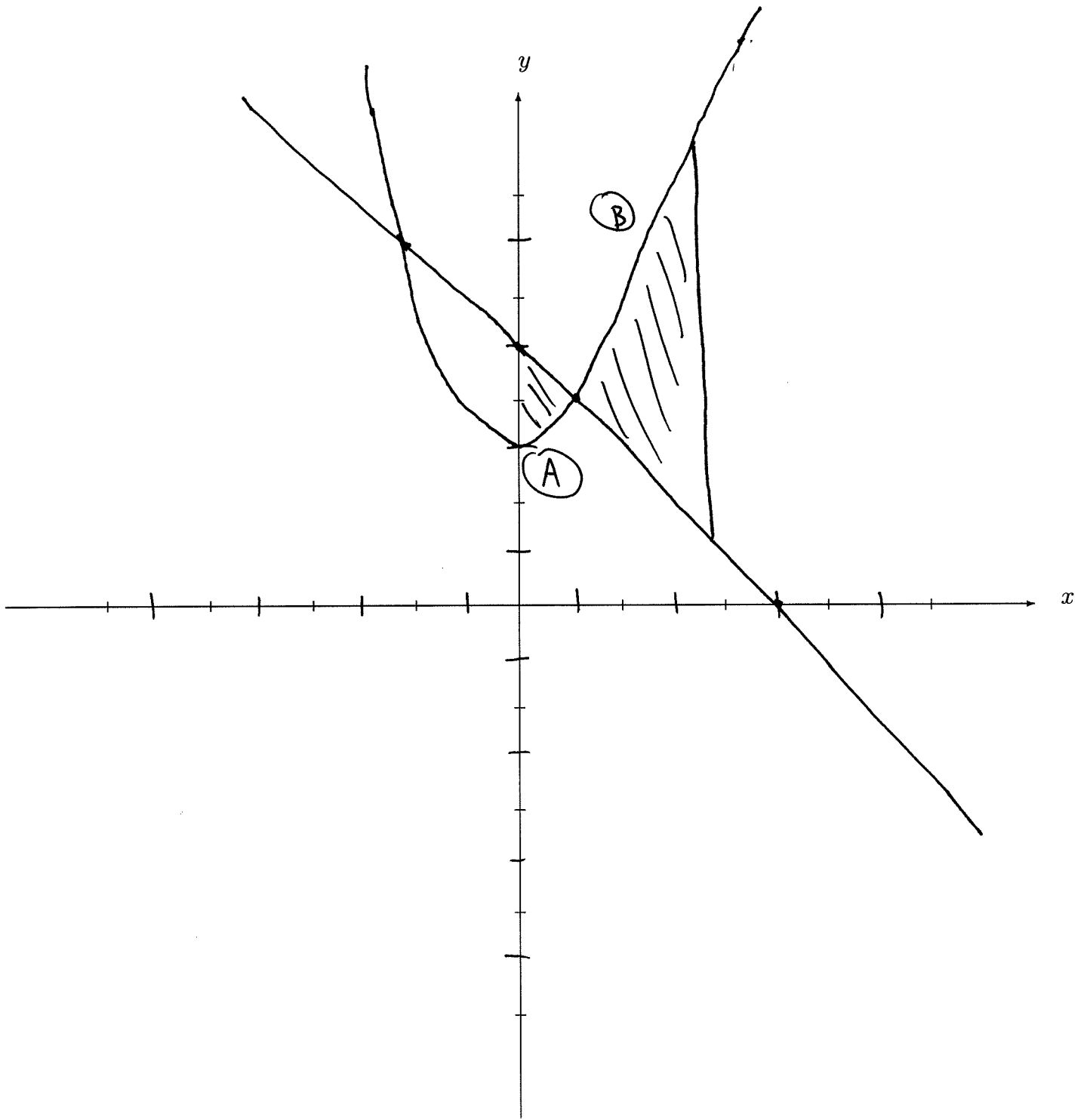
So IP, at $(-2, 7)$ & $(1, 4)$

$$\text{Area of (A)} = \int_0^1 [(5-x) - (3+x^2)] dx = \int_0^1 (-x^2 - x + 2) dx$$
$$= \left[-\frac{x^3}{3} - \frac{x^2}{2} + 2x \right]_0^1 = -\frac{1}{3} - \frac{1}{2} + 2 = \frac{7}{6}$$

$$\text{Area of (B)} = \int_1^4 [(3+x^2) - (5-x)] dx = \int_1^4 (x^2 + x - 2) dx$$

$$= \left[\frac{x^3}{3} + \frac{x^2}{2} - 2x \right]_1^4 = \left(\frac{64}{3} + 8 - 8 \right) - \left(\frac{1}{3} + \frac{1}{2} - 2 \right)$$
$$= \frac{135}{6}$$

$$\text{Total Area} = \frac{7}{6} + \frac{135}{6} = \frac{142}{6} = \frac{71}{3}$$



Long Answer Question 5 (12 points)

Consider the function of two variables

$$f(x, y) = x^2 - 4x + 6xy + 2y^3 - 6.$$

- (a) (3 points) Calculate the first-order partial derivatives.
- (b) (3 points) Find all critical points.
- (c) (6 points) Identify what type of critical points they are (local max, local min or saddle point).

$$f_x = 2x - 4 + 6y = 0 \Rightarrow 2x + 6y = 4$$

$$f_y = 6x + 6y^2 = 0 \Rightarrow x = -y^2$$

Combining the 2, $\frac{-2y^2 + 6y = 4}{y^2 - 3y + 2 = 0}$

$$y^2 - 3y + 2 = 0$$

$$(y - 2)(y - 1) = 0$$

So $y = 1$ or 2 .

So CP's at $(-1, 1)$ & $(-4, 2)$

$$\left. \begin{array}{l} f_{xx} = 2 \\ f_{yy} = 12y \\ f_{xy} = 6 \end{array} \right\} \Rightarrow D = 2 \cdot (12y) - 6^2 = 24y - 36$$

So, for $(-1, 1)$, $D = -12$. It's a saddle point

for $(-4, 2)$, $D = 12$. Since $f_{xx} > 0$,

we have a local min