THE UNIVERSITY OF CALGARY, FACULTY OF SCIENCE
MATHEMATICS 271 L01, L02
FINAL EXAMINATION, WINTER 2015
TUESDAY, APRIL 28, 2015 FROM 12:00 TO 3:00

LAST NAME________________ FIRST NAME____________ ID____________

LECTURE NUMBER________________ INSTRUCTOR NAME____________

EXAMINATION RULES

1. This is a closed book examination.
2. No aids are allowed for this examination.
3. The use of personal electronic or communication devices is prohibited.
4. A University of Calgary Student ID card is required to write the Final Examination and could be requested for midterm examinations. If adequate ID isn't present the student must complete an Identification Form.

Question	Total Marks	Actual Marks
1	7	
2	9	
3	10	
4	13	
5	12	
6	12	
7	9	
8	8	
Total	80	

[7] 1. (a) Use the Euclidean algorithm to find $\gcd(94, 49)$. Also use the algorithm to find integers u and v such that $\gcd(94, 49) = 94u + 49v$.

(b) Use part (a) to find an inverse a for 49 modulo 94 so that $0 \leq a \leq 93$; that is, find an integer $a \in \{0, 1, \ldots, 93\}$ so that $49a \equiv 1 (\bmod 94)$.

[9] 2. Let $\mathcal{P}$ be the statement:

"For all sets A, B and C, if $A - B \subseteq A - C$ then $A \cap C = \emptyset$."

(a) Is $\mathcal{P}$ true? Prove your answer.

(b) State the *converse* of $\mathcal{P}$. Is the converse of $\mathcal{P}$ true? Prove your answer.

(c) State the *contrapositive* of $\mathcal{P}$. Is the contrapositive of $\mathcal{P}$ true? Explain.

[10] 3. Of the following statements, two are true and one is false. Prove the true statements. For the false statement, write out its negation and prove that.

(a) For all integers a, there exists an integer b so that $3 \mid a + b$.

(b) For all integers a, there exists an integer b so that $3 \nmid a + b$.

(c) For all integers a, there exists an integer b so that $3 \mid a + b$ and $3 \mid 2a + b$.

[13] 4. Recall that $\mathbb{Z}$ is the set of all integers.
(a) Find a function $f : \mathbb{Z} \to \mathbb{Z}$ so that f is one-to-one and onto. Explain why f is one-to-one and onto.

(b) Find a function $g : \mathbb{Z} \to \mathbb{Z}$ so that g is one-to-one but g is not onto. Explain why g is one-to-one but g is not onto.

(c) Find a function $h : \mathbb{Z} \to \mathbb{Z}$ so that h is onto but h is not one-to-one. Explain why h is onto but h is not one-to-one.

[12] 5. Let $A = \{1, 2, 3, 4, ..., 271\}$. Define the relation $\mathcal{R}$ on $A \times A$ by:

$$\text{for any } (a, b)\,,\ (c, d) \in A \times A, \qquad (a, b)\,\mathcal{R}\,(c, d) \text{ if and only if } a + b = c + d.$$

(a) Prove that $\mathcal{R}$ is an equivalence relation on $A \times A$.

(b) List all the elements of $[(3, 3)]$, the equivalence class of $(3, 3)$.

(c) How many equivalence classes does $\mathcal{R}$ have? Explain.

(d) Is there an equivalence class that has exactly 271 elements? Explain.

[12] 6. Let $S = \{1,2,3,4,5,6,7,8,9\}$. Define a relation $\mathcal{R}$ on $\mathcal{P}(S)$ by:

$$\text{for any } X, Y \in \mathcal{P}(S), \qquad X \mathcal{R} Y \text{ if and only if } X \cap Y \neq \emptyset.$$

(a) Is $\mathcal{R}$ reflexive? symmetric? transitive? Explain.

(b) Let $A = \{1,2\}$. Find three different subsets $I,\ J,\ K$ of S so that $I\mathcal{R}A$, $J\mathcal{R}A$ and $K\mathcal{R}A$.

(c) Let $A = \{1,2\}$. How many subsets X of S are there so that $X\mathcal{R}A$? Explain.

[9] 7. (a) Let G be a simple graph with vertex degrees 2, 2, 2, 2, 3, 3. How many edges does G have? Explain.

(b) Is there a simple graph G with vertex degrees 2, 2, 2, 2, 3, 3 so that G has an Euler circuit? Explain.

(c) Draw, if possible, a simple bipartite graph G with vertex degrees 2, 2, 2, 2, 3, 3 or explain why no such graph exists.

(d) Are there simple graphs G and H both with vertex degrees 2, 2, 2, 2, 3, 3 such that G and H are not isomorphic? If so, draw them; otherwise, explain why such graphs do not exist.

[8] 8. Prove by induction on n that $n^3 - 7n$ is divisible by 3 for all integers $n \geq 0$.