MATH 3705C
Test 1 Solutions
February 3, 2017

[Marks]

[4] 1. $\mathcal{L}\left\{e^{-3t}\cos(2t)\right\} =$

(a) $\dfrac{s-3}{(s-3)^2+4}$ (b) $\dfrac{s}{(s+3)^2+4}$ (c) $\dfrac{s+3}{(s+3)^2+4}$ (d) $\dfrac{s+3}{s^2+4}$ (e) None of these

Answer: (c)

[4] 2. If $f(t) = \left\{\begin{array}{cc} \sin(3t-6), & t \geq 2 \\ 0, & t < 2 \end{array}\right\}$, then $\mathcal{L}\{f(t)\} =$

(a) $\dfrac{e^{-2s}}{s^2+9}$ (b) $\dfrac{3}{(s-2)^2+9}$ (c) $\dfrac{3e^{-2s}}{s^2+9}$ (d) $\dfrac{3e^{-6s}}{s^2+9}$ (e) None of these

Answer: (c)

[4] 3. $\mathcal{L}^{-1}\left\{\dfrac{2s+9}{s^2-s-6}\right\} =$

(a) $3e^{3t} - e^{-2t}$ (b) $3e^{-3t} - e^{2t}$ (c) $e^{-2t} - 3e^{3t}$ (d) $e^{2t} - 3e^{-3t}$ (e) None of these

Answer: (a)

[4] 4. $\mathcal{L}^{-1}\left\{\dfrac{(s+1)e^{-2s}}{s^2+4s+13}\right\} =$

(a) $u(t-2)e^{-2(t-2)}\left\{\cos[3(t-2)] - \sin[3(t-2)]\right\}$

(b) $u(t-2)e^{-2(t-2)}\left\{\cos[3(t-2)] - \frac{1}{3}\sin[3(t-2)]\right\}$

(c) $u(t-2)e^{-2(t-2)}\left\{\cos[3(t-2)] - 3\sin[3(t-2)]\right\}$

(d) $u(t-2)e^{-(t-2)}\cos[3t-6]$

(e) None of the above

Answer: (b)

[7] 5. Employ the Laplace transform to solve the initial-value problem
$y'' + y' - 2y = 0,\ y(0) = 1,\ y'(0) = 2.$

Solution:

$$s^2Y(s) - s - 2 + sY(s) - 1 - 2Y(s) = 0 \ \Rightarrow\ Y(s) = \frac{s+3}{s^2+s-2} = \frac{s+3}{(s-1)(s+2)}$$

$$= \frac{4}{3}\frac{1}{s-1} - \frac{1}{3}\frac{1}{s+2} \ \Rightarrow\ y(t) = \frac{4}{3}e^{t} - \frac{1}{3}e^{-2t}.$$

[7] 6. Employ the Laplace transform to solve the initial-value problem
$y'' - 2y' + 5y = \delta(t-3),\ y(0) = 0,\ y'(0) = 1.$

Solution:

$$s^2Y(s) - 1 - 2sY(s) + 5Y(s) = e^{-3s} \ \Rightarrow\ Y(s) = \frac{1+e^{-3s}}{s^2-2s+5} = \frac{1+e^{-3s}}{(s-1)^2+4} \ \Rightarrow$$

$$y(t) = \frac{1}{2}e^{t}\sin(2t) + \frac{1}{2}u(t-3)e^{t-3}\sin[2(t-3)].$$