University of Ottawa

MAT 2377 – Midterm II, Winter 2018

March 22, 2018 **Professor: Chen Xu and Termeh Kousha**
Time: 80 minutes

Student Number:__________

Family Name : __________ **First Name:** ____________________

This is a closed book examination. Only basic calculators are allowed. Record your answer to each of the multiple choice questions in the table below (NOT for Question 7 and 8).

Grade:———/**15**

Question	Answer	Your Grade
1	B	
2	A	
3	C	
4	E	
5	F	
6	D	
7		
8		

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You MUST sign below
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Multiple Choice Questions (1 marks/question for a total 6 marks)

Please Record your answer to each multiple choice question in the table on the first page.

1. A random sample of 50 suspension helmets used by a motorcycle riders and automobile race-car drivers was subjected to an impact test, and on 18 of these helmets some damage was observed. Find a 90% confidence interval for the true proportion of helmets of this type that would show damage on this test.

   A) [0.27, 0.49] B) [0.25, 0.47] C) [0.19, 0.50] D) [0.23, 0.49]
   E) [0.21, 0.49]
   Answer B

   Solution:

   A point estimate for the true proportion of helmets of this type that would show damage on this test is

$$\widehat{p} = \frac{x}{n} = \frac{18}{50} = 0.36.$$

The estimated standard error for the estimate in part (a) is

$$\widehat{\sigma}_{\widehat{P}} = \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} = \sqrt{\frac{.36(1-.36)}{50}} = 0.0679$$

A 90% confidence interval for the true proportion of helmets of this type that would show damage on this test is

$$\begin{aligned} & \widehat{p} \pm z_{0.05}\sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} \\ = \;& .36 \pm 1.645\,(0.0679) \\ = \;& .36 \pm 0.1117 \\ = \;& [0.25, 0.47] \end{aligned}$$

2. The Greenland ice sheet covers roughly 80% of the surface of Greenland, being the second largest body of ice in the world, after the Antarctic ice sheet. As the arctic climate is rapidly warming, the Greenland ice sheet has experienced record melting in the recent years. The following data gives the depth of the ice sheet (in m) measured at various locations during the summer months in the Northeast Greenland National Park. We displayed the data in below

   3125 3115 3118 3145 3123 3131 3124 3133 3127 3120

   Compute the sample median of this random sample. Is the median a measure of central tendency or of dispersion?

   A) median = 3124.5, measure of central tendency
   B) median = 3123, measure of central tendency
   C) median = 3124.5, measure of dispersion
   D) median = 3126.1, measure of central tendency
   E) median = 3126.1, measure of dispersion
   F) None of the answers

   Answer A

   Solution: First we sort the data in ascending order:

   3115 3118 3120 3123 3124 3125 3127 3131 3133 3145

   $n = 10$, therefore:

   $$(1 - 0.5)y_5 + 0.5\, y_6 = 0.5(3124) + 0.5(3125) = 3124.5.$$

   The median is a measure of central tendency.

3. The carbon dioxide (CO2) emission of the latest model of Toyota Prius is a random variable following a normal distribution with mean $\mu = 70$ g/km and standard deviation $\sigma = 1.5$ g/km. Find x_0 such that 93% of

these vehicles have an emission level smaller than x_0.

A) 71.4 B) 73.5 C) 72.2 D)72.9 E)70.8

Answer: C

Solution: By standardization, we have:

$$0.93 = P(X < x_0) = P\left(\frac{X-70}{1.5} < \frac{x_0-70}{1.5}\right) = P\left(Z < \frac{x_0-70}{1.5}\right).$$

From Table 17.3, we find $z_0 = 1.475$ such that $P(Z < z_0) = 0.93$.

$$\frac{x_0-70}{1.5} = 1.475.$$

Hence $x_0 = 70+(1.5)(1.475) = 72.2125$. The answer is C. The incorrect answer A is obtained using $z_0 = 1.96$.

4. A machine produces metal rods used in an automobile suspension system. A random sample of 8 rods is selected, and the diameter is measured. The resulting data (in millimeters) are as follows:

8.24	8.25	8.20	8.23
8.21	8.26	8.26	8.28

The sample mean is $\overline{x} = 8.24125$ and the sample standard deviation is $s = 0.02696$. Assume that the diameter of this type of metal rod follows a Normal distribution. Calculate a 95% confidence interval for the mean rod diameter.

A)[8.15, 8.36] B) [7.98, 8.26] C) [8.19, 8.27] D) [8.21, 8.23]
E) [8.22, 8.26]

Answer E

Solution:

A 95% confidence interval for μ is

$$\begin{aligned} & \overline{x} \pm t_{.025,8-1} \frac{s}{\sqrt{n}} \\ = & \; 8.24125 \pm 2.365 \left(\frac{0.02696}{\sqrt{8}} \right) \\ = & \; 8.24125 \pm 0.02254 \\ = & \; [8.22, 8.26] \end{aligned}$$

5. Let $X_1 \sim N(2, 1^2)$ and $X_2 \sim N(1, 2^2)$ be two normal random variables. Assume that X_1 and X_2 are independent. Let $Y = 3X_1 - X_2$. What is the value of $sd(Y)$?

A)2.236 B) 1 C) 5 D) -1 E) 2.646 F) 3.606

Answer F

Solution:

Since $Y = 3X_1 - X_2$, we have

$$Var(Y) = 3^2 Var(X_1) + (-1)^2 Var(X_2) = 9 * 1 + 4 = 13.$$

Thus, $sd(Y) = \sqrt{Var(Y)} = \sqrt{13} = 3.606$.

6. The following data show the length (in cm) of 12 randomly selected white shrimps in a grocery store.

$$x_1 = 4.2 \quad x_2 = 4.6 \quad x_3 = 4.3 \quad x_4 = 4.9 \quad x_5 = 4.6 \quad x_6 = 4.5$$

$$x_7 = 4.2 \quad x_8 = 4.7 \quad x_9 = 4.9 \quad x_{10} = 4.4 \quad x_{11} = 5.1 \quad x_{12} = 5.6$$

Which statement is true?

A) The IQR is 0.412 and 4.2 and 5.6 are the outliers.
B) The IQR is 0.575 and 5.6 is an outlier.
C) The IQR is 0.575 and 4.2 and 5.6 are the outliers.
D) The IQR is 0.575 and there are no outliers.
E) The IQR is 0.412 and 5.6 is an outlier.
F) The IQR is 0.412 and there are no outliers.

Solution

The answer is D). We need first to sort the data:

$$x_1 = 4.2 \quad x_2 = 4.6 \quad x_3 = 4.3 \quad x_4 = 4.9 \quad x_5 = 4.6 \quad x_6 = 4.5$$

$$x_7 = 4.2 \quad x_8 = 4.7 \quad x_9 = 4.9 \quad x_{10} = 4.4 \quad x_{11} = 5.1 \quad x_{12} = 5.6$$

$n = 12$, $\frac{n+1}{4} = 3 + \frac{1}{4}$. The first quartile is

$$Q_1 = (0.75)y_3 + (0.25)y_4 = (0.75)(4.3) + (0.25)(4.4) = 4.325.$$

$\frac{3(n+1)}{4} = 9 + \frac{3}{4}$.The third quartile is

$$Q_3 = (0.25)y_9 + (0.75)y_{10} = (0.25)(4.9) + (0.75)(4.9) = 4.9.$$

Thus, $IQR = Q_3 - Q_1 = 4.9 - 4.325 = 0.575$.

The fences are located at $Q_1 - 1.5(IQR) = 4.325 - 0.8625 = 3.4625$ and $Q_3 + 1.5(IQR) = 4.9 + 0.8625 = 5.7625$. Since all values are within the fences [3.4625, 5.7625], there is no outlier in this data set.

Short answer questions (total of 9 marks)

7. (4 points) (a) The amount of time that a customer spends waiting at an airport check-in counter is a random variable with mean 8.5 minutes and standard deviation 1.5 minutes. Suppose that a random sample of $n = 49$ customers is observed. Approximate the probability that the average time waiting in line for these customers is between 5 and 10 minutes.

   (b) Suppose that in part (a) we wanted the error in estimating the mean of the waiting time to be at most half a minute (30 seconds) at 95% confidence. What sample size should be used?

   Solution:
   (a) Let $\overline{X}$ be the average waiting time of the $n = 49$ customers. Since n is large, then by the C.L.T., the sample mean $\overline{X}$ is approximately normally distributed. Its mean is $\mu_{\overline{X}} = \mu = 8.5$ and its standard deviation is $\sigma_{\overline{X}} = \sigma/\sqrt{n} = 1.5/\sqrt{49} = 0.21429$. The probability that the average time waiting in line for these customers is between 5 and 10 minutes is

$$\begin{aligned} P(5 \leq \overline{X} \leq 10) &\approx \Phi\left(\frac{10-8.5}{0.21429}\right) - \Phi\left(\frac{5-8.5}{0.21429}\right) \\ &= \Phi(7.0) - \Phi(-16.33) \approx 1 - 0 = 1 \end{aligned}$$

   (b) Solving

$$n = \left(\frac{z_{.025}\,\sigma}{E}\right)^2 = \left(\frac{1.96\,(1.5)}{1/2}\right)^2 = 34.57.$$

   We require $n = 35$ observations.

   Marking Scheme 2 points for each part (a) and (b)

8. (5 points) Assume that arrivals of small aircraft at a particular airport can be modeled as a Poisson process with a mean rate of 1 aircraft per hour.

(a) Determine the length of an interval of time (in hours) such that the probability that no arrivals occur during the interval is 0.10.

(b) What is the probability that the waiting time for 3 arrivals will be more than 3 hours?

(c) What is the mean and variance for the waiting time for 3 arrivals?

Solution: (a) Let X be the waiting time for an arrival in hours. X follows an exponential distribution with $\lambda = 1$. We want to find x such

$$0.1 = P(X > x) = 1 - P(X \leq x) = 1 - (1 - e^{-\lambda x}) = e^{-x}.$$

Hence $x = -\ln(.1) = 2.3026$ hours.

(b) Let X be the waiting time (in hours) for 3 arrivals. X follows an Erlang distribution with $\lambda = 1$ and $r = 3$. We want

$$\begin{aligned} P(X > 3) &= 1 - P(X \leq 3) = 1 - \left[1 - \sum_{k=0}^{r-1} \frac{e^{-\lambda(3)}[\lambda(3)]^k}{k!}\right] \\ &= 1 - \left[1 - \left(\frac{e^{-3}3^0}{0!} + \frac{e^{-3}3^1}{1!} + \frac{e^{-3}3^2}{2!}\right)\right] = 1 - 0.5768 = 0.4232 \end{aligned}$$

(c) Let X be the waiting time (in hours) for 3 arrivals. X follows an Erlang distribution with $\lambda = 1$ and $r = 3$. We want

$$\mu_X = \frac{r}{\lambda} = \frac{3}{1} = 3 \quad \text{and} \quad \sigma_X^2 = \frac{r}{\lambda^2} = \frac{3}{1^2} = 3.$$

Marking Scheme 2 points part (a) and (b), 1 point each for part (c) (half a point each)

Cumulative distribution function for $N(0,1)$:$\Phi(z) = P(Z \leq z)$

0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.00	z
.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	−3.8
.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	−3.7
.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002	−3.6
.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	−3.5
.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	−3.4
.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005	−3.3
.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007	−3.2
.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010	−3.1
.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013	−3.0
.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019	−2.9
.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026	−2.8
.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035	−2.7
.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047	−2.6
.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062	−2.5
.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082	−2.4
.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107	−2.3
.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139	−2.2
.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179	−2.1
.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228	−2.0
.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287	−1.9
.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359	−1.8
.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446	−1.7
.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548	−1.6
.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668	−1.5
.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808	−1.4
.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968	−1.3
.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151	−1.2
.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357	−1.1
.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587	−1.0
.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841	−0.9
.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119	−0.8
.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.242	−0.7
.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743	−0.6
.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085	−0.5
.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446	−0.4
.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821	−0.3
.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207	−0.2
.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602	−0.1
.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000	−0.0

Cumulative distribution function for $N(0,1)$:$\Phi(z) = P(Z \leq z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

The T distribution with ν degrees of freedom

	$F_T(t) = P(T \leq t)$						
	.6	.75	.9	.95	.975	.99	.995
ν	$t_{.40,\nu}$	$t_{.25,\nu}$	$t_{.10,\nu}$	$t_{.05,\nu}$	$t_{.025,\nu}$	$t_{.01,\nu}$	$t_{.005,\nu}$
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012
14	0.258	0.692	1.345	1.761	2.145	2.624	2.997
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787
26	0.256	0.684	1.315	1.706	2.056	2.479	2.779
27	0.256	0.684	1.314	1.703	2.052	2.473	2.771
28	0.256	0.683	1.313	1.701	2.048	2.467	2.763
29	0.256	0.683	1.311	1.699	2.045	2.464	2.756
30	0.256	0.683	1.310	1.697	2.042	2.457	2.750
∞	0.253	0.674	1.282	1.645	1.96	2.326	2.576

∞

Extra page for calculations