Practice Test 2 for MAT 2377 (Winter 2018)
Probability and statistics for engineers (solutions)

Please enter your choices for the multiple choice questions in the following table.

Question	Answer
1	A
2	D
3	D
4	C
5	C
6	A

Short Answer Questions

[4] 1. Our old plastic moulding machine produces parts with a mean breaking strength of $\mu = 1000$ kilograms. A new model has come out which promises to increase the mean breaking strength. Money is tight, so the boss will only authorize the purchase if there are significant evidence that the claims about the new model are true. We can run some trials with the new machine but it is very expensive, so we can only a do a few. They produce eight plastic parts with the new machine and the eight breaking strengths are

$$1156,\ 1279,\ 1144,\ 1141,\ 1261,\ 1247,\ 963,\ 1094.$$

[1] (a) Give point estimates for the mean breaking strength and the standard deviation of the breaking strength.

Solution: We start by computing the following sums:

$$\sum_{i=1}^{n} x_i = 9,285.0 \quad \text{and} \quad \sum_{i=1}^{n} x_i^2 = 10,852,129.0.$$

A point estimate for the mean breaking strength is

$$\hat{\mu} = \overline{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{9,285.0}{8} = 1160.625 \text{ kg}$$

and a point estimate for the standard deviation of the breaking strength is

$$\hat{\sigma} = s = \sqrt{\frac{(\sum_{i=1}^{n} x_i^2) - (\sum_{i=1}^{n} x_i)^2/n}{n-1}}$$
$$= \sqrt{\frac{10,852,129.0 - (9,285.0)^2/8}{7}} = 104.0095 \text{ kg.}$$

[1] (b) Give the estimated standard error of the estimate of the mean breaking strength.

Solution: The estimated standard error of the estimate of the mean breaking strength is

$$\hat{\sigma}_{\overline{X}} = \frac{s}{\sqrt{n}} = \frac{104.0095}{\sqrt{8}} = 36.7729 \text{ kg.}$$

[2] (c) Assuming that the breaking strength is normally distributed, compute a 95% confidence interval for the mean breaking strength.

solution:

A 95% confidence interval for the mean breaking strength is

$$\overline{x} \pm t_{0.025;7}\frac{s}{\sqrt{n}} = 1160.625 \pm (2.365)(36.7729)$$
$$= [1073.657, 1247.593].$$

[4] 2. Suppose that X has the following probability density function:

$$f(x) = (1/3)\, e^{-x/3}, \quad x > 0.$$

(a) Let $\mu = E[X]$ and $\sigma = \sqrt{V[X]}$. Give the values of μ and σ.

(b) Compute $P(X > 5 | X > 3)$.

(c) Suppose that we collect a random sample $X_1, X_2, \ldots, X_{35}$ from a population with the above density. Let

$$\overline{X} = \frac{\sum_{i=1}^{35} X_i}{35}.$$

Approximate $P(2.75 < \overline{X} < 3.25)$.

Solution:

(a) X has an exponential distribution with $\lambda = 1/3$. Thus,

$$\mu = E[X] = \frac{1}{\lambda} = 3 \quad \text{and} \quad \sigma = \sqrt{V[X]} = \sqrt{\frac{1}{\lambda^2}} = 3.$$

(b) The exponential satisfies the memoryless property, so

$$P(X > 5|X > 3) = P(X > 2) = e^{-\lambda(2)} = e^{-2/3} = 0.5134.$$

(c) Since $n = 35$ is large, by the Central Limit Theorem, then

$$\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) = N(3, 9/35) \quad \text{approximately.}$$

We want

$$\begin{aligned} P(2.75 < \overline{X} < 3.25) &\approx \Phi\left(\frac{3.25 - 3}{\sqrt{9/35}}\right) - \Phi\left(\frac{2.75 - 3}{\sqrt{9/35}}\right) \\ &= \Phi(0.49) - \Phi(-0.49) \\ &= 0.6879 - 0.3121 \\ &= 0.3758 \end{aligned}$$

Multiple Choice Questions

Please enter your answers to the multiple choice question in the table provided on the first page.

[1] 1. Earthquakes in a given region follow a Poisson process with rate 0.05 per month. What is the probability that we will have to wait more than 12 months for 2 earthquakes?

A) 0.8781 B) 0.1219 C) 0.3292 D) 0.0988 E) 0.9012

Solution: Let T_2 the waiting time for 2 earthquakes in months. We want

$$P(T_2 > 12) = P(N(12) \leq 1) = e^{-0.6}\frac{0.6^0}{0!} + e^{-0.6}\frac{0.6^1}{1!} = 0.8781,$$

where $N(1)$ is the number of earthquakes in 12 months and $N(1)$ has a Poisson distribution with mean $\lambda t = 0.05(12) = 0.6$.

[1] 2. Let X be a normally distributed random variable with mean μ and standard deviation $\sigma = 17.5$. Determine the size of the sample required to be 90% confident that the error in the estimation of μ is at most 5.3.

A) 125 B) 75 C) 15 D) 30 E) 42

Solution: Solve

$$n \geq \left[\frac{z_{0.05}\,\sigma}{E}\right]^2 = \left[\frac{(1.645)(17.5)}{5.3}\right]^2 = 29.50.$$

We require $n = 30$ observations.

[1] 3. We have a random sample of size 10 from a normal population with mean of 5. Let $\overline{X}$ and S be the sample mean and the sample standard deviation, respectively. Find a value c such that

$$P\left(\frac{\overline{X} - 5}{S/\sqrt{10}} \leq c\right) = 0.025.$$

A) 2.262 B) 2.228 C) −1.96 D) −2.262 E) −2.228

Solution: The random variable $(\overline{X} - 5)/(S/\sqrt{10})$ has a $t(9)$ distribution. Thus, $c = t_{0.975,9} = -t_{0.025,9} = -2.262$.

[1] 4. The diameter of a certain bullet is known to have a normal distribution. From a random sample of size 10, we compute a mean diameter of 1.50 cm and a standard deviation of 0.01 cm. Find a 99% confidence interval for the mean diameter.

A) $1.45 \leq \mu \leq 1.55$
B) $1.47 \leq \mu \leq 1.53$
C) $1.49 \leq \mu \leq 1.51$
D) $1.39 \leq \mu \leq 1.61$
E) $1.46 \leq \mu \leq 1.54$

Solution: A 99% confidence interval for the mean diameter is

$$\overline{x} \pm t_{0.005,9}\frac{s}{\sqrt{n}} = 1.5 \pm (3.250)\frac{0.01}{\sqrt{10}} = [1.49; 1.51].$$

[1] 5. Engineers want to test the production capacity of a particular type of wind turbines. Assume that the mean amount of power provided by a single turbine is $\mu = 120$ kWh and that the standard deviation is $\sigma = 20$ kWh. Suppose 50 turbines are put in 50 independent locations, what is the approximate probability that the average power of the 50 turbines will be more than 125 kWh?

(A) 0.5987 (B) 0.2660 (C) 0.0384 (D) 0.4913 (E) 0.9616

Solution: Let $\overline{X}$ the average power of the 50 turbines. By the Central Limit Theorem, we have

$$\frac{\overline{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1) \text{ approximately.}$$

Then,

$$\begin{aligned} P(\overline{X} > 125) &\approx 1 - \Phi\left(\frac{125-120}{20/\sqrt{50}}\right) \\ &= 1 - \Phi(1.77) = 1 - 0.9616 = 0.0384. \end{aligned}$$

[1] 6. The strength of a steel alloy is normally distributed with a mean 20 gigapascals (GPa) and a standard deviation of 3.54 GPa. Determine a value x such that 25% of the specimens have a strength (in GPa) greater than x.

A) 22.4 B) 17.6 C) 23.0 D) 17.0 E) 20.0

Solution: We want x such that $0.25 = P(X > x)$. Equivalently, we want x such that $0.75 = P(X \leq x) = P(Z \leq (x-20)/3.54)$. We get

$$0.67 = \frac{x-20}{3.54} \quad \Rightarrow \quad x = (0.67)(3.54) + 20 = 22.3718 \approx 22.4.$$