1. **(10 marks)** According to the Movie Theatre Association of Canada (MTAC), the average price for a movie ticket in Canada in 2016 was $10.30. Assume that the population standard deviation is $0.65. Suppose that a sample of 30 theatres was randomly selected.

   a. **(1 mark)** Calculate the mean of the sampling distribution of $\bar{x}$.

   $\mu_{\bar{x}} = \mu = \$10.30$

   b. **(2 marks)** Calculate the standard deviation of the sampling distribution of $\bar{x}$.

   $$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.65}{\sqrt{30}} = 0.119$$

   c. **(2 marks)** __Briefly__ describe the sampling distribution of $\bar{x}$.

   The sampling distribution of $\bar{x}$ will be approximately normally distributed with mean $= \$10.30$ and Standard deviation $= \$0.119$

   d. **(2 marks)** What is the probability that the sample mean will be more than $10?

   $$P(\bar{x} > 10) = P\left(z > \frac{10-10.3}{0.119}\right) = P(z > -2.52) = F(2.52)$$

   $$= 0.9941$$

   e. **(3 marks)** The probability is 0.24 that the sample mean is less than what number?

   $P(\bar{x} < a) = 0.24 \Rightarrow P(z < b) = 0.24 \Rightarrow b = -0.71$

   $\Rightarrow \bar{x} = z\frac{\sigma}{\sqrt{n}} + \mu = (-0.71)(0.119) + 10.3$

   $\Rightarrow \bar{x} = \$10.216$

2. **(9 marks)** It has been found that summer electricity bills for single-family homes in Montreal follow a normal distribution with standard deviation \$100. A random sample of 25 bills was taken.

a. **(3 marks)** Find the probability that the sample standard deviation is less than \$75.

$P(S<75) = P(S^2 < 5625)$

$\chi^2_{n-1} = \frac{(n-1)S^2}{\sigma^2} \Rightarrow P\left(\chi^2_{24} < \frac{24(5{,}625)}{10{,}000}\right) = P(\chi^2_{24} < 13.5)$

$P(\chi^2_{24} > 12.401 = 0.975)$; $P(\chi^2_{24} > 13.848) = 0.95$

$\Rightarrow 0.950 < P(\chi^2_{24} > 13.5) < 0.975 \Rightarrow 0.025 < P(\chi^2_{24} < 13.5) < 0.05$

$\therefore\ 0.025 < P(S<75) < 0.05$

b. **(3 marks)** Find the probability that the sample standard deviation is more than \$120.

$P(S>120) = P(S^2 > 14{,}400)$

$P\left(\chi^2_{24} > \frac{24(14{,}400)}{10{,}000}\right) = P(\chi^2_{24} > 34.56)$

$P(\chi^2_{24} > 33.196) = 0.10$; $P(\chi^2_{24} > 36.415) = 0.05$

$\Rightarrow 0.05 < P(\chi^2_{24} > 3.196) < 0.10$

$\Rightarrow 0.05 < P(S>120) < 0.10$

c. **(3 marks)** The probability is 0.05 that the sample standard deviation is bigger than what number?

$P(S^2 > K) = P\left(\frac{(n-1)K}{\sigma^2} > \chi^2_{n-1}\right) = 0.05 \Rightarrow P\left(\frac{24K}{10{,}000} > 36.415\right) = 0.05$

$\Rightarrow \frac{24K}{10{,}000} = 36.415 \Rightarrow K = \frac{(36.415)(10{,}000)}{24} = 15{,}172.917$

$\Rightarrow S^2 = K = 15{,}172.917 \Rightarrow S = \sqrt{K} = \sqrt{15{,}172.917} = 123.178$

$\Rightarrow S = \$123.178$

3. **(10 marks)** Direct mail advertisers send "junk mail" to thousands of potential customers in the hope that some will buy the company's product; the response rate is usually quite low. Suppose a company wants to test the response to a new flyer and sends it to 1000 people, they get orders from 123 of the recipients.

   a. **(2 marks)** Calculate the margin of error for a 95 percent confidence interval for the true proportion of recipients who may buy something after receiving the flyer.

$$ME = z\sqrt{\frac{p(1-p)}{n}}\,; \quad 95\%\ C.I. \Rightarrow z_{0.025} = 1.96$$

$$\Rightarrow ME = 1.96\sqrt{\frac{(0.123)(0.877)}{1000}} = 1.96(0.010) = 0.020$$

   b. **(2 marks)** Construct a 95 percent confidence interval for the true proportion of recipients who may buy something after receiving the flyer.

$$C.I. = \hat{p} \pm ME = 0.123 \pm 0.020 = (0.103, 0.143)$$

   c. **(2 marks)** **Briefly** interpret the confidence interval you found in part (b).

We are 95% confident that the proportion of recipients who may buy something lies between 10.3% and 14.3%

   d. **(2 marks)** **Briefly** explain what "95% confidence" means.

If we were to repeatedly take samples of size 1,000 and construct confidence intervals in a similar manner, then 95% of such intervals would contain the true proportion.

e. **(2 marks)** Suppose the company wanted to be 95 percent confident that its estimate is within 0.05 of the true proportion, calculate the required sample size.

$$n = \frac{z^2(0.25)}{ME^2} = \frac{(1.96)^2(0.25)}{0.05^2} = 384.16$$

round <u>up</u> to <u>385</u>

4. **(15 marks)** The Canadian air traffic control system handled an average of 14,577 flights per day during 30 randomly selected days. The standard deviation for this sample is 1,895 flights per day.

a. **(2 marks)** Calculate the margin of error for a 99% confidence interval for the true mean number of flights handled per day by the system.

$$ME = t_{29, 0.005} \frac{s}{\sqrt{n}} = 2.756\left(\frac{1895}{\sqrt{30}}\right)$$

$$= 953.516$$

b. **(2 marks)** Construct a 99% confidence interval for the true mean number of flights handled per day by the system.

$$C.I. = \bar{x} \pm ME = 14,577 \pm 953.516$$

$$= (13,623.484, 15,530.516)$$

c. **(2 marks)** **<u>Briefly</u>** explain what assumptions were necessary to construct the confidence interval in parts (a) and (b).

We need to assume that an ~~a~~ independent random sample was selected from a normally distributed population.

d. **(2 marks)** Suppose that the current system can safely handle 20,000 flights per day. **Briefly** explain whether we can conclude that the current system needs an upgrade.

Since the current average lies between 13,623.484 and 15,530.516, with 99 % confidence, then there is no need to upgrade the system.

e. **(2 marks)** Suppose you wanted to be 90% confident, would the margin of error be larger or smaller than the one found in part (a)? **Briefly** explain.

With 90 % confidence, ME would be smaller because the critical value would be smaller; with a lower confidence level, we can have a narrower interval.

f. **(5 marks)** Construct a 95% confidence interval for the true population standard deviation of the number of flights per day.

$$\frac{(n-1)s^2}{\chi^2_{n-1,\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{n-1,1-\alpha/2}} \quad \Bigg| \quad \begin{array}{l} \chi^2_{29,0.025} = 45.722 \\ \chi^2_{29,0.975} = 16.047 \end{array}$$

$$\Rightarrow \frac{29(1895^2)}{45.722} < \sigma^2 < \frac{29(1895^2)}{16.045}$$

$$2,277,672.127 < \sigma^2 < 6,490,478.342$$

$$1,509.196 < \sigma < 2,547.642$$

5. **(8 marks)** A random sample of 574 Canadian undergraduate Economics students was selected and the students were asked whether or not they were users of Twitter. Of the 280 females in the sample, 74 reported being Twitter users compared while 62 of the 294 males reported that they use Twitter. Let p_1 and p_2 represent the proportion of females and males, respectively, who use Twitter.

a. **(3 marks)** Calculate the margin of error for a 95% confidence interval for $p_1 - p_2$.

$\hat{p}_1 = \frac{74}{280} = 0.264$; $\hat{p}_2 = \frac{62}{294} = 0.211$

$$ME = z_{0.025}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

$$= 1.96\sqrt{\frac{0.264(0.736)}{280} + \frac{0.211(0.789)}{294}}$$

$$= 1.96(0.035) = 0.069$$

b. **(3 marks)** Construct a 95% confidence interval for $p_1 - p_2$.

$$C.I. = (\hat{p}_1 - \hat{p}_2) \pm ME$$

$$= (0.264 - 0.211) \pm 0.069$$

$$= 0.053 \pm 0.069$$

$$= (-0.016, 0.122)$$

c. **(2 marks)** Based on the interval in part (b), briefly explain whether we can conclude that the proportion of female students who use Twitter exceeds the proportion of male students.

Since the interval contains zero, at the 95% level of confidence, we cannot be confident that the proportion of females who use twitter exceeds the proportion of males.

6. **(13 marks)** A company implements an exercise break for its workers to see if this would improve job satisfaction, as measured by a questionnaire that assesses workers' satisfaction. Scores for 6 randomly selected workers before and after implementation of the exercise programme are shown below.

Worker Number	Before	After	d_i	$(d_i - \bar{d})^2$
1	34	33	-1	72.25
2	28	36	8	0.25
3	29	50	21	182.25
4	15	21	6	2.25
5	45	41	-4	132.25
6	24	39	15	56.25
				445.5

a. **(2 marks)** Calculate the sample mean difference in the job satisfaction scores.

$$\bar{d} = \frac{\Sigma d_i}{n} = \frac{45}{6} = 7.5$$

b. **(3 marks)** Calculate the standard deviation of the difference in the job satisfaction scores.

$$S_d^2 = \frac{\Sigma(d_i - \bar{d})^2}{n-1} = \frac{445.5}{5} = 89.1$$

$$\Rightarrow S_d = \sqrt{S_d^2} = \sqrt{89.1} = 9.439$$

c. **(3 marks)** Calculate the margin of error for a 98% confidence interval for the true difference in job satisfaction scores before and after implementation of the exercise break (that is, $after - before$).

$$ME = t_{5,0.01}\left(\frac{S_d}{\sqrt{n}}\right) = 3.365 \sqrt{\frac{9.439}{\sqrt{6}}}$$

$$ME = 3.365(3.853)$$
$$= 12.965$$

d. **(3 marks)** Construct a 98% confidence interval for the true difference in job satisfaction scores before and after implementation of the exercise break.

$C.I. = \bar{X}_d \pm ME = 7.5 \pm 12.965$

$= (-5.465, 20.465)$

e. **(2 marks)** <u>**Briefly**</u> explain whether we can conclude that the exercise break improved job satisfaction scores.

Since the interval includes zero, we cannot conclude, at the 98% confidence level, that the programme improved job satisfaction scores.

End of exam. Good luck!