

Assignment 1 due tomorrow (Friday 29th)

submit SHARP @ 3:00 PM

Drop box, MAT Dep 585 KED
(Not to ME :))

Review

Random Variable: $X: \mathcal{S} \rightarrow \mathbb{R}$ $X \rightarrow$ discrete
 $\searrow$ continuous

Discrete R.V.

$$f_X(x) = P(X=x) \quad \text{p.m.f}$$

$$\sum_{x \in R_X} f_X(x) = 1$$

$$F_X(x) = P(X \leq x) \quad \text{c.d.f}$$

$$R_X = \{-1, 0, 1/2\}$$

$$F_X(1) = P(X=-1) + P(X=0) + P(X=1)$$

$$= f_X(-1) + f_X(0) + f_X(1)$$

$$\text{Ex 1: } f(x) = \begin{cases} 0.72 & x=0 \\ 0.26 & x=1 \\ 0.02 & x=2 \end{cases}$$

Expected value: (measure of Central tendency)

Weighted average of the possible values taken by X $\rightarrow$ If you repeat the experiment many times $\rightarrow$ then you expect the value of X to be approximately $E(X)$.

$$\mu = E(X) = \sum_{x \in R_X} x f(x)$$

In Ex 1:

$$E(X) = 0 \cdot f(0) + 1 \cdot f(1) + 2 \cdot f(2)$$

$$= 0 \cdot (0.72) + 1 \cdot (0.26) + 2 \cdot (0.02) =$$

$$M = 0.3$$

If you roll a fair die:

$$\mathcal{S} = \{1, 2, 3, 4, 5, 6\}$$

$$P(X=1) = P(X=2) = \dots = P(X=6)$$

$$= \frac{1}{6}$$

$$E(X) = 1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} = 3.5$$

Sample space



$$(x - \mu)^2$$

$$\text{Variance } \text{Var}(X) = \sigma^2$$

(measure of dispersion)

On average how far we are going to be from the center.

Expected value

$$\text{Var}(X) = \sigma^2 = E[(X - \mu)^2] =$$

$$\sum_{x \in R_X} (x - \mu)^2 f(x)$$

$$= E(X^2) - E^2(X)$$

$$= E(X^2) - \mu^2$$

$$\text{Note: } E(X^2) = \sum_{x \in R_X} x^2 f(x)$$

$$\text{In general, if } E(h(X)) = \sum_{x \in R_X} h(x) f(x)$$

Standard deviation: (σ) SD

Square root of the Variance

$$\sigma = \sqrt{\text{Var}(X)}$$

Ex 1.

$$E(X) = \mu = 0.3 \quad f(x) = \begin{cases} 0.72 & x=0 \\ 0.26 & x=1 \\ 0.02 & x=2 \end{cases}$$

$$\text{Var}(X) = E(X^2) - E^2(X) = E(X^2) - \mu^2$$

$$\sum x^2 f(x) - \mu^2 = [(0)^2(0.72) + (1)^2(0.26) + (2)^2(0.02)] - (0.3)^2 = 0.25$$

or,

$$\sigma^2 = \text{Var}(X) = E[(X - \mu)^2] = \sum_{x \in R_X} (x - \mu)^2 f(x)$$

$$= (0 - 0.3)^2 0.72 + (1 - 0.3)^2 0.26 + (2 - 0.3)^2 0.02 =$$

$$0.25$$

$$\text{SD}(X) = \sigma \Rightarrow \sqrt{\text{Var}(X)} = \sqrt{0.25} = 0.5$$

Ex 3. Roll a die

$$P(X=1) = \dots = P(X=6)$$

$$= \frac{1}{6}$$

$$E(X) = \mu = 3.5$$

$$\text{Var}(X) = E(X^2) - \mu^2 = \left[(1)^2 \frac{1}{6} + (2)^2 \frac{1}{6} + (3)^2 \frac{1}{6} + (4)^2 \frac{1}{6} + (5)^2 \frac{1}{6} + (6)^2 \frac{1}{6} \right] - (3.5)^2 = \frac{35}{12}$$

A player pays \$1 per game to play.

The player wins \$3 with probability $\frac{1}{3}$ and wins nothing with probability $\frac{2}{3}$.If X is the value of the game

(value of the game - won - paid)

Compute $E(X)$ X = value of the game,

$$\text{win: } X = 3 - 1 = 2 \quad R_X = \{-1, 2\}$$

$$\text{lose: } X = 0 - 1 = -1 \quad P(X=2) = \frac{1}{3}$$

$$P(X=-1) = \frac{2}{3}$$

$$E(X) = \sum x f(x) = (-1)(\frac{2}{3}) + 2(\frac{1}{3}) = 0$$

Linear Transformation

$$Y = a + bX \quad a, b \in \mathbb{R}$$

$$E(Y) = a + b E(X)$$

$$E(a + bX)$$

$$\text{Var}(Y) = \text{Var}(a + bX) = b^2 \text{Var}(X)$$

$$\text{SD}(Y) = |b| \text{SD}(X)$$

$$\text{Ex 1. Continued: } E(X) = 0.3, \text{Var}(X) = 0.25, Y = C(X) = 125 + 25X$$

$$E(C(X)) = 125 + 25E(X) = 132.5$$

$$\text{Var}(C(X)) = (25)^2 \cdot \text{Var}(X) = (25)^2 (0.25)$$

$$\text{Ex: } Y = -2 - 0.2X$$

$$\mu_Y = E(Y) = -2 - 0.2 E(X)$$

$$\text{Var}(Y) = (-0.2)^2 \text{Var}(X) = 0.04 \sigma_X^2$$

$$\sigma_Y = \text{SD}(Y) = |-0.2| \text{SD}(X) = 0.2 \sigma_X$$

Continuous R.V.

Range of X is an Interval X = height of the students

$$P(X = 175.9) = 0 \quad (100, 220)$$

$$A = (170, 175)$$

$$P(A) = \frac{2}{3}$$

Cumulative distribution function

$$F_X(x) = P(X \leq x)$$

 $f(x)$ = density function $F_X(b)$ = Area under the curve $f(x)$

$$= \int_{-\infty}^b f(x) dx$$

The area under the curve is 1

$$\int_{-\infty}^{\infty} f(x) dx = 1$$

$$F_X(b) = P(X \leq b)$$

$$= P(X < b)$$

$$P(a \leq X \leq b) = P(a < X < b) = P(a \leq X < b) = P(a < X \leq b)$$

$$= \int_a^b f(x) dx$$

$$= F_X(b) - F_X(a)$$

$$f_X'(x) = \frac{d}{dx} F_X(x)$$

$$\text{Ex 4. a) } f_X(x) = \begin{cases} 0 & x < 0 \\ x/2 & 0 \leq x \leq 2 \\ 0 & x > 2 \end{cases}$$

density

$$\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^0 0 dx + \int_0^2 x/2 dx + \int_2^{\infty} 0 dx$$

$$= \int_0^2 x/2 dx = 1$$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ x^2/4 & 0 \leq x \leq 2 \\ 1 & x > 2 \end{cases}$$

$$\star \int_0^x f(z) dz = \int_0^x \frac{z}{2} dz = \frac{z^2}{4} \Big|_0^x = \frac{x^2}{4}$$

$$P(0.5 < X < 1.5) = F_X(1.5) - F_X(0.5)$$

$$= (1.5)^2/4 - (0.5)^2/4$$

$$P(X = 1) = 0 = 1/2$$



$$P(X > 2) = 1 - P(X \leq 2)$$

$$= 1 - F_X(2) = 1 - 1 = 0$$

$$\mu = E(X) = \int_{-\infty}^{\infty} x f_X(x) dx$$

$$E(h(X)) = \int_{-\infty}^{\infty} h(x) f_X(x) dx$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x) dx$$

$$\text{Var}(X) = E(X^2) - E^2(X)$$

$$= E(X^2) - \mu^2$$

$$\text{Ex 4. b) } f_X(x) = \begin{cases} \lambda \exp(-\lambda x) & x > 0 \\ 0 & x \leq 0 \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ 1 - e^{-\lambda x} & x > 0 \end{cases}$$

$$F_X(x) = \int_{-\infty}^x \lambda e^{-\lambda z} dz = \int_{-\infty}^0 0 dz + \int_0^x \lambda e^{-\lambda z} dz$$

$$= \lambda \int_0^x e^{-\lambda z} dz = \lambda \left(-\frac{1}{\lambda} \right) e^{-\lambda z} \Big|_0^x = -e^{-\lambda x} - (-e^0)$$

$$= 1 - e^{-\lambda x}$$

$$\text{Linear transformation (continuous) } Y = a + bX \quad \text{Var}(Y) = b^2 \text{Var}(X)$$

$$E(Y) = a + b E(X) \quad \text{SD}(Y) = |b| \text{SD}(X)$$