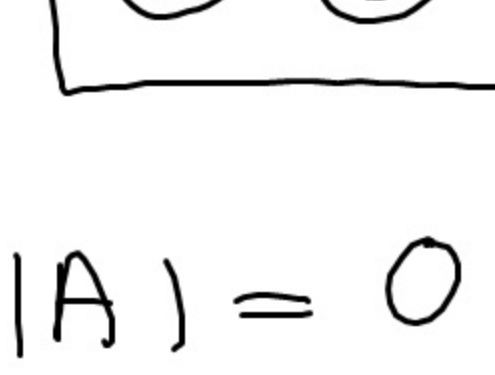


Review

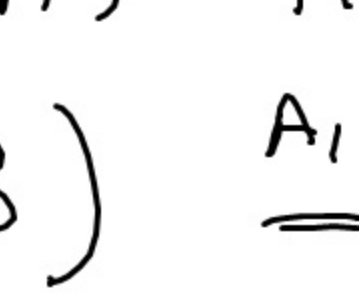
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

A, B are Mutually Exclusive. $(A \cap B = \emptyset)$



$$P(A|B) = 0 \quad P(B|A) = 0$$

A and B are Ind. $P(A), P(B) \neq 0$



$$P(A|B) = P(A) \quad P(B|A) = P(B)$$

$$P(A \cap B) = P(A) P(B) \quad A, B \text{ ind.} \Rightarrow A', B' \text{ ind.}$$

DeMorgan

$$P(A \cup B) = 1 - P(A' \cap B') = 1 - P(A') P(B') = 1 - (1 - P(A))(1 - P(B))$$

Chapter 2 : Random Variables (RV)

Roll a die $S = \{1, 2, 3, 4, 5, 6\}$

D_1 = "event that the outcome is 1"

D_2 = " " " " " " " 2 "

D_6 = " " " " " " " 6 "

D_7 = " " " " " " " is at most 2 "

RV X is a function that assigns a real number to the outcomes.

$$X: S \rightarrow \mathbb{R}$$

R_X : range X

Flip a coin $\{H, T\}$ $\rightarrow$ Prob. of the event

$$X = \{0, 1\} \quad P(X=0) \text{ the outcome is head}$$

$$P(X=1) \text{ Prob. " " is Tail.}$$

A RV X is a variable whose numerical value is determined by chance.

Rolling a die $R_X = \{1, 2, 3, 4, 5, 6\}$

D_1 "event that outcome is one" $X=1$

D_7 " " " " " is at most 2 " $X \leq 2$

Random Variables $\rightarrow$ Discrete

$\rightarrow$ Continuous.

EX: a couple plan to have 3 children. X = # of girls.

R_X = Set of all possible numbers

Range of X that we can observe

$$\text{Discrete. Finite} \leftarrow R_X = \{0, 1, 2, 3\}$$

Random Variables we always

use Capital letters X, Y, Z

Y = # of siblings a student may have

$$\leftarrow R_Y = \{0, 1, 2, 3, \dots\}$$

Discrete - infinite.

W = height of the students in the class. (in cm)

Continuous Random Variables $R_W = (100, 200)$ Continuous.

$$174.12 \quad \text{Interval} \quad \begin{array}{c} 100 \quad 200 \end{array}$$

When W is a Cont RV then

$$P(W = 164) = 0 \quad \text{any constant}$$

Discrete Random variables

R_X = Discrete (finite, infinite)

Probability Mass Function PMF

$$f_X(x) = P(X=x) \quad x \in R_X$$

Fair coin $R_X = \{0, 1\}$

$$f(0) = P(X=0) = \frac{1}{2}$$

$$f(1) = P(X=1) = \frac{1}{2}$$

Roll a die $R_X = \{1, 2, \dots, 6\}$ $f(2) = P(X=2) = \frac{1}{6}$

$$\textcircled{1} 0 \leq f_X(x) \leq 1$$

$$\textcircled{2} \sum_{x \in R_X} f_X(x) = 1$$

A Cumulative Distribution Function (CDF)

the probability to observe at most x

$$F_X(x) = \sum_{Y \leq x} f_Y$$

Roll a die

$$F_X(2) = P(X=1) + P(X=2) = \frac{2}{6}$$

$$P(X \leq 2)$$

EX 1. 2 Components that are Ind.

D_1 = "event that the first component is Defective"

D_2 = " " " " " second " "

$$P(D_1) = 0.1 \quad P(D_2) = 0.2$$

X = # of defective Components

$$\text{a) } R_X = \{0, 1, 2\}$$

b) P.m.f

$$P(X=0) = P(D_1' \cap D_2') = P(D_1') P(D_2') = (1 - P(D_1))(1 - P(D_2)) = 0.9 \cdot 0.8 = 0.72$$

$$P(X=1) = P(D_1' \cap D_2) + P(D_1 \cap D_2')$$

$$\stackrel{\text{Ind}}{=} P(D_1') P(D_2) + P(D_1) P(D_2')$$

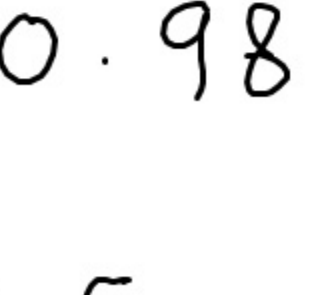
$$= (0.9)(0.2) + (0.1)(0.8) = 0.26$$

$$P(X=2) = P(D_1 \cap D_2) = P(D_1) P(D_2) = (0.1)(0.2) = 0.02$$

Pmf

$$f_X(x) = \begin{cases} 0.72 & X=0 \\ 0.26 & X=1 \\ 0.02 & X=2 \end{cases}$$

$$\sum_{x \in R_X} f_X(x) = f_X(0) + f_X(1) + f_X(2) = 1$$

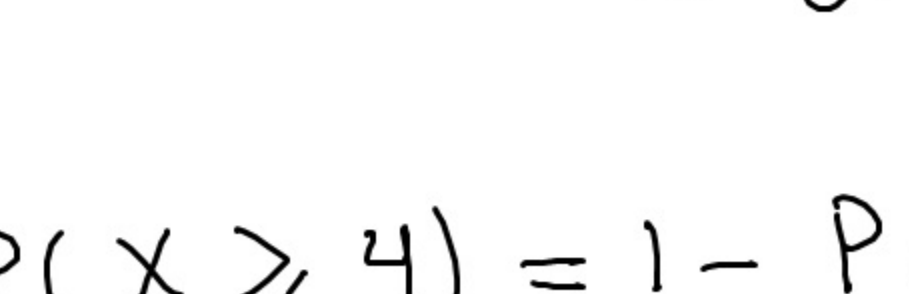


c) C d f

$$F_X(x) = \begin{cases} 0 & X < 0 \\ P(X=0)=0.72 & 0 \leq X < 1 \\ P(X=0)+P(X=1)=0.98 & 1 \leq X < 2 \\ P(X=0)+P(X=1)+P(X=2)=1 & X \geq 2 \end{cases}$$

$$\lim_{x \rightarrow -\infty} F_X(x) = 0 \quad \lim_{x \rightarrow \infty} F_X(x) = 1$$

$F_X(x)$ is increasing.



$$\text{d) at least 1 } f_X(1) + f_X(2)$$

$$P(X \geq 1) = P(X=1) + P(X=2) = 0.28$$

or

$$\rightarrow 1 - P(X < 1) = 1 - P(X=0) = 1 - 0.72 = 0.28$$

$$\text{e) } P(X > 2) = 1 - P(X \leq 2) = 1 - F_X(2) = 1 - 1 = 0$$

$$\text{f) } P(-0.2 < X \leq 1.5) = P(X=0) + P(X=1) = 0.98$$



$$P(a \leq X \leq b) = P(X \leq b) - P(X \leq a)$$



$$P(-0.2 < X \leq 1.5) = P(X \leq 1.5) - P(X \leq -0.2) = F_X(1.5) - F_X(-0.2)$$

$$= 0.98 - 0 = 0.98$$

EX 2.

x	0	1	2	3	4	5
$f(x)$	0.81	0.07	0.05	0.04	0.02	0.01

X = # of persons per day who are see this online necessarily

$$\text{i) } P(X \geq 4) = P(X=4) + P(X=5) = 0.02 + 0.01 = 0.03$$

$$\text{Alternatively } P(X \geq 4) = 1 - P(X < 4) = 1 - (P(X=0) + P(X=1) + P(X=2) + P(X=3)) = 1 - 0.93 = 0.07$$

$P(X \leq x)$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ f_X(0)=P(X=0)=0.81 & 0 \leq x < 1 \\ P(X=0)+P(X=1)=0.88 \text{ (} F_X(0)+P(X=1) \text{)} & 1 \leq x < 2 \\ P(X=0)+P(X=1)+P(X=2)=0.93 \text{ (} F_X(1)+P(X=2) \text{)} & 2 \leq x < 3 \\ P(X=0)+P(X=1)+P(X=2)+P(X=3)=0.97 \text{ (} F_X(2)+P(X=3) \text{)} & 3 \leq x < 4 \\ P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)=0.99 \text{ (} F_X(3)+P(X=4) \text{)} & 4 \leq x < 5 \\ P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)=1 \text{ (} F_X(4)+P(X=5) \text{)} & x \geq 5 \end{cases}$$

Try to get $f_X(x)$ from $F_X(x)$