MAT 2377 – Lecture 4 – Jan 18th 2018

Conditional Probability

$P(B) \neq 0$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

equally likely events

$P(A|B) = \frac{\#(A \cap B)}{\#(B)}$

$$P(A|B)P(B) = P(A \cap B)$$

$P(A) \neq 0$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \qquad P(B|A)P(A) = P(A \cap B)$$

$$\boxed{P(A|B)P(B) = P(B|A)P(A)}$$

③ $P(\phi|B) = 0$

② $A \subseteq C \quad P(A|B) \leq P(C|B)$

④ $0 \leq P(A|B) \leq 1$

$P(A'|B) =$ $1 - P(A|B)$

Note: A and B are Mutually exclusive ($A \cap B = \phi$). If one occurs, the other one can't occur.

$$P(A|B) = 0 \quad P(B|A) = 0$$

$\hookrightarrow \frac{P(A \cap B)}{P(B)} \overset{0}{=} 0$

$$P(B) = P(B \cap A) + P(B \cap A') = P(B|A)P(A) + P(B|A')P(A')$$

$P(B) = P(B \cap E_1) + \cdots + P(B \cap E_k) = P(B|E_1)P(E_1) + \cdots + P(B|E_k)P(E_k)$

Bayes' Theorem:

$$P(E_i|B) = \frac{P(E_i \cap B)}{P(B)} = \frac{P(B|E_i)P(E_i)}{P(B|E_1)P(E_1) + \cdots + P(B|E_k)P(E_k)}$$

Ex 14.

M = "event the sample has Norvag" $P(M) = 0.32$

L = " " " " Lead" $P(L) = 0.16$

$P(M \cup L) = 0.38 = P(M) + P(L) - P(L \cap M)$

$0.38 = 0.32 + 0.16 - P(L \cap M) \Rightarrow P(L \cap M) = 0.1$

1. $P(L|M) = \frac{P(L \cap M)}{P(M)} = \frac{0.1}{0.32} = 0.31$

2. A = "event ..... arsenic"

$P(A|M) = 0.45$ (among the ones -)

$P(A \cap M) = P(A|M)P(M) = 0.45(0.32) = 0.144$

3. $P(A|M') = 0.02$

$A \nearrow M$, $A \nearrow M'$

$P(A) = P(A|M)P(M) + P(A|M')P(M')$ (Partition) $1 - P(M)$

$= (0.45)(0.32) + (0.02)(0.68)$

$= 0.1576$

Independent Events & A and B.

Two events A and B are ind. if they have no influence on each other.

① $P(B|A) = P(B)$

② $P(A|B) = P(A)$

③ $P(A \cap B) = P(A)P(B)$

(These three are equivalent).

① → ③ : $P(B|A) = P(B)$

$\frac{P(A \cap B)}{P(A)} = P(B) \Rightarrow P(A \cap B) = P(A)P(B)$ (ind)

$E_1, \ldots, E_k$ are independent events if $P(E_1 \cap E_2 \cap \cdots \cap E_k) = P(E_1)P(E_2) \cdots P(E_k)$

A and B are ind

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= P(A) + P(B) - P(A)P(B)$

A and B are ind $\Leftrightarrow P(A \cap B) = P(A)P(B)$

$\Leftrightarrow P(A|B) = P(A)$ & $P(B|A) = P(B)$

② A and B' are also Ind. $P(A \cap B') = P(A)P(B')$

③ A' and B " " " $P(A' \cap B) = P(A')P(B)$

④ A' and B' " " " $P(A' \cap B') = P(A')P(B')$

Ex 16. number of out comes

$N(S) = 20$, $N(A) = 5$, $N(B) = 10$, $N(A \cap B) = 3$

If A and B are ind?

$P(A) = \frac{N(A)}{N(S)} = \frac{5}{20}$ $\quad P(B) = \frac{10}{20}$

$P(A \cap B) = \frac{3}{20}$

If A and B are ind $\rightarrow P(A \cap B) = P(A)P(B)$

Check $P(A)P(B) = \frac{5}{20} \cdot \frac{10}{20} = \frac{1}{8} \neq \frac{3}{20} = P(A \cap B)$

They are not Ind.

Ex 17. $N(S) = 20$ $N(A) = 5$ $N(B) = 12$ $N(A \cap B) = 3$

$P(A) = \frac{5}{20}$ $\quad P(B) = \frac{12}{20}$ $\quad P(A \cap B) = \frac{3}{20}$

$P(A \cap B) \overset{?}{=} P(A)P(B)$

$\frac{3}{20} \overset{\checkmark}{=} \frac{5}{20} \cdot \frac{12}{20} = \frac{1}{4} \times \frac{3}{5} = \frac{3}{20}$

A and B are Ind.

Ex 18

A_i = "event that the ith component will work" $\quad i = 1, \ldots, 5$

$P(A_i) = 0.9$ $\quad A_1, A_2, \ldots, A_5$ are ind

a) All 5 will work:

$P(A_1 \cap A_2 \cap \cdots \cap A_5) = P(A_1) \cdots P(A_5)$ and ind

$(0.9) \cdot (0.9) = (0.9)^5 = 0.59$

b) at least 1

De Morgan

$P(A_1 \cup A_2 \cup \cdots \cup A_5) = 1 - P(A_1' \cap \cdots \cap A_5')$

When the ones are ind the ones are ind

$= 1 - P(A_1')P(A_2') \cdots P(A_5')$

$= 1 - (0.1)^5 = 0.9999$ $\quad P(A_i') = 1 - P(A_i) = 0.1$

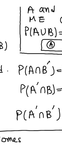
c) Only third will work:

$P(A_1' \cap A_2' \cap A_3 \cap A_4' \cap A_5')$

$= P(A_1')P(A_2')P(A_3)P(A_4')P(A_5')$

$= (0.1)(0.1)(0.9)(0.1)(0.1) = 9 \times 10^{-4}$

Ex 19



C7 If either C_1 or C_2 works, we have a path.

C8 C_3 or C_4

C9 C_5 or C_6

But for the circuit to be functional C_7, C_8, and C_9 they must all work.

All the circuits are working independently from each other.

C_i = the event that the ith circuit is functional.

$P(C_7) = P(C_1 \cup C_2)$

C_1, C_2 are ind $= 1 - P(C_1' \cap C_2') = 1 - P(C_1')P(C_2') = 1 - (0.02)(0.1) = 0.998$

$P(C_8) = P(C_3 \cup C_4) = 1 - P(C_3' \cap C_4') = 1 - P(C_3')P(C_4') = 1 - (0.03)(0.1) = 0.997$

$P(C_9) = P(C_5 \cup C_6) = 1 - P(C_5' \cap C_6') = 1 - P(C_5')P(C_6') = 1 - (0.05)(0.1) = 0.995$

0.998 (C7) 0.997 (C8) 0.995 (C9)

In order for the system to be functional C_7 and C_8 and C_9 must work:

All events are ind

$P(C_7 \cap C_8 \cap C_9) = P(C_7)P(C_8)P(C_9)$

$= 0.998 \cdot 0.997 \cdot 0.995 = 0.99$