

Review

Probability $\begin{cases} \rightarrow \text{Subjective} \\ \rightarrow \text{Classic (equally likely)} \\ \rightarrow \text{Relative frequency model} \end{cases}$

$$0 \leq P(E) \leq 1 \quad P(\emptyset) = 0 \quad P(S) = 1$$

$$E_1, E_2, \dots \quad E_i \cap E_j = \emptyset \quad i \neq j$$

$$ME \quad P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

$$P(E_1 \cup \dots \cup E_n) = P(E_1) + \dots + P(E_n)$$

$$A \cap A' = \emptyset \quad A \cup A' = S$$

$$P(S) = P(A \cup A') = 1$$

$$P(A) + P(A') = 1 \quad P(A') = 1 - P(A)$$

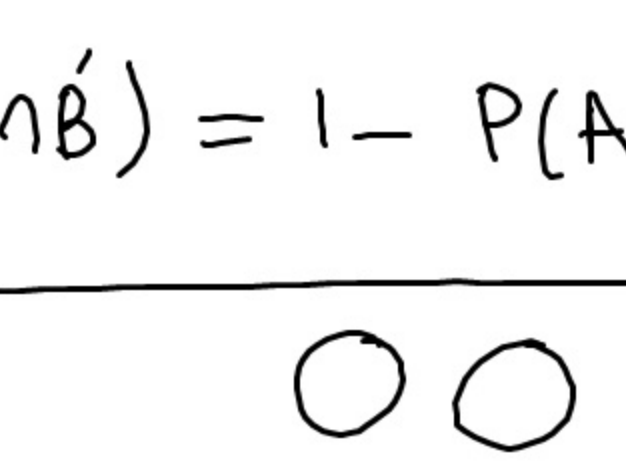
$$\star P(A) = P(\underline{A \cap B}) + P(\underline{A \cap B'}) \quad \boxed{\text{Venn Diagram}}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$P(E_1 \cup \dots \cup E_n) = 1 - P(E_1' \cap \dots \cap E_n')$$

$$P(E_1 \cap \dots \cap E_n) = 1 - P(E_1' \cup \dots \cup E_n')$$

EX  $N(S) = 20$ $N(A) = 10$ $N(B) = 5$ $N(A \cap B) = 2$ $P(A) = \frac{10}{20} = \frac{1}{2}$ $P(B) = \frac{5}{20} = \frac{1}{4}$ $P(A \cap B) = \frac{2}{20} = \frac{1}{10}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{10}{20} + \frac{5}{20} - \frac{2}{20} = \frac{13}{20}$$

$$P(A' \cap B') = 1 - P(A \cup B) = 1 - \frac{13}{20} = \frac{7}{20}$$

EX  Red Blue Pink White Black

randomly choose 3 balls. What is the prob that we choose Red / Pink / Black.

$$\frac{1}{5C3} = \frac{1}{\binom{5}{3}} = \frac{1}{\frac{5!}{2!3!}} = 0.1$$

(b) the probability that first I pick red - then pink then black.

$$\frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{5P_3}$$

EX 11.

E: event that a piece of integrated circuit has a defective etching

$$P(E) = 0.12$$

S: event that a " " " " has a defective slot

$$P(S) = 0.29$$

$$P(E \cap S) = 0.07 \quad \text{and} \quad P(E) = P(E \cap S) + P(E \cap S')$$

a) $P(E \cap S') = P(E) - P(E \cap S) = 0.12 - 0.07 = 0.05$
 But-and $\cap$ but

b) $P(E \cup S) = P(E) + P(S) - P(E \cap S) = 0.12 + 0.29 - 0.07 = 0.34$
 or DeMorgan Law

c) $P(E' \cap S') = 1 - P(E \cup S) = 1 - 0.34 = 0.66$

Group of 100 German

70% Blond hair

77% Blue eyes

55% Blond and Blue hair eyes

B: event Blond hair

E: event Blue eyes



$$P(B) = \frac{70}{100}$$

$$P(E) = \frac{77}{100}$$

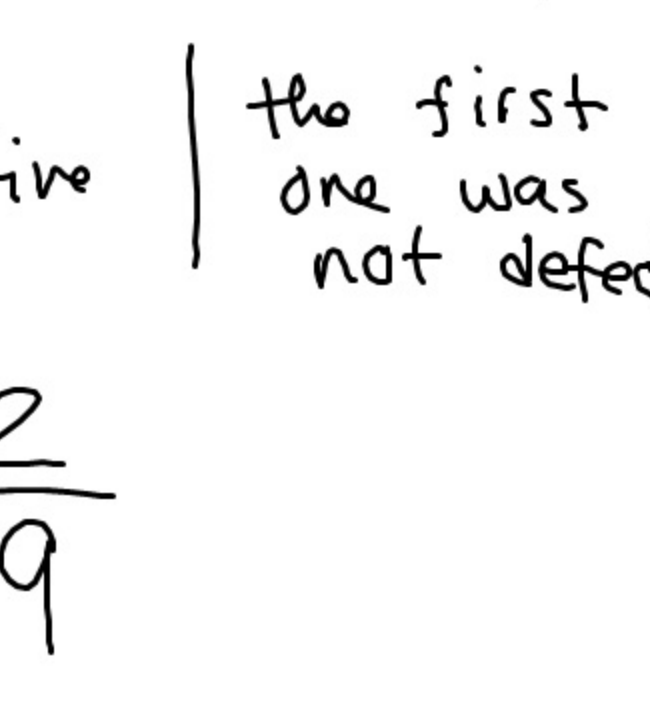
$$P(B \cap E) = \frac{55}{100}$$

Select randomly a man in the group which have a blond hair. What is the probability that the man has also blue eyes.

$$P(E | B) = \frac{55}{70}$$

given

$$\frac{P(A \cap B)}{P(B)}$$



$$P(A | B) = \text{Probability that A occurs knowing that the outcome must be in B}$$

$$P(B) \neq 0$$

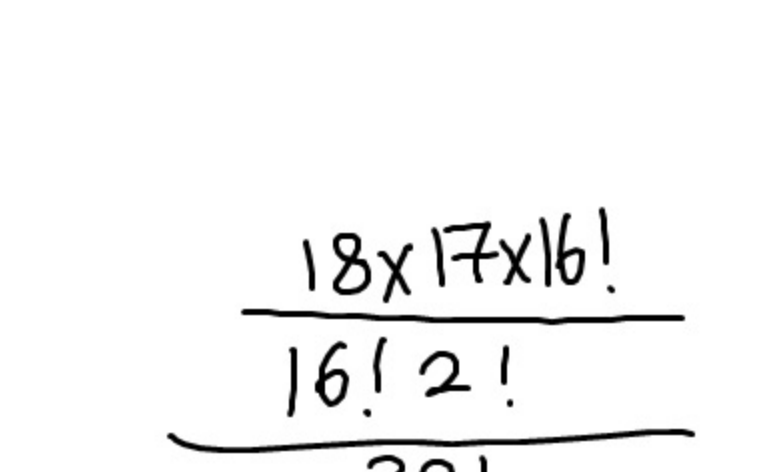
$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

Multiplication Rule $P(A \cap B) = P(A | B) P(B)$
 $\rightarrow P(B \cap A)$

$P(A) \neq 0$ $P(B | A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = P(B | A) P(A)$

$$P(A | B) P(B) = P(B | A) P(A)$$

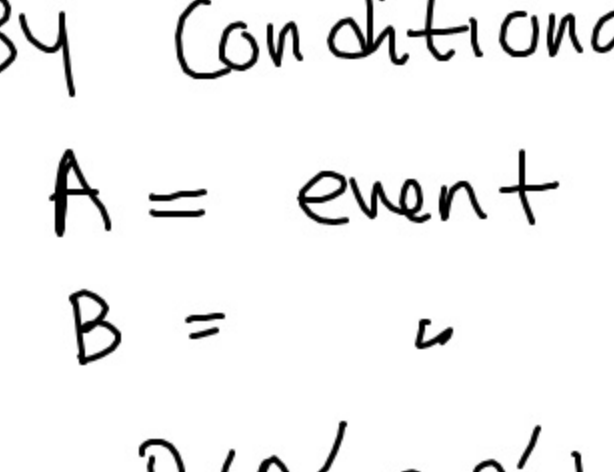
EX 12. Choose 2 by random (without replacement)



$$P(\text{second one is defective} | \text{the first one was not defective}) = \frac{2}{19}$$

EX 13.

Choose 2



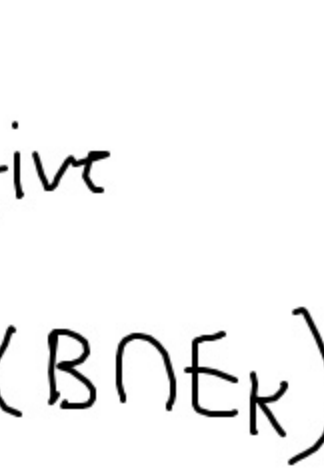
$$P(\text{Both are not defective}) = \frac{18C2}{20C2} = \frac{\binom{18}{2}}{\binom{20}{2}}$$

$$= \frac{18 \times 17 \times 16!}{20!} = \frac{18 \times 17}{20 \times 19}$$

By Conditional probability:

A = event the first one is defective.

B = " " " second " " "

by multiplication Rule $P(A' \cap B') = P(B' | A') P(A')$
 $\frac{17}{19} \cdot \frac{18}{20} = \frac{17 \times 18}{19 \times 20}$
 

Total probability Rule

$$P(B) = P(B \cap A) + P(B \cap A')$$

$$P(B) = P(B | A) P(A) + P(B | A') P(A')$$



$$E_i \cap E_j = \emptyset \quad i \neq j \quad ME$$

$$E_1 \cup \dots \cup E_k = S \quad \text{exhaustive}$$

$$P(B) = P(B \cap E_1) + \dots + P(B \cap E_k)$$

$$P(B) = P(B | E_1) P(E_1) + \dots + P(B | E_k) P(E_k)$$

$$B \rightarrow E_1$$

$$B \rightarrow E_2$$

$$B \rightarrow E_k$$

Bayes' Theorem

$E_1, \dots, E_k$ ME + Exhaustive events.

$$P(E_i | B) = \frac{P(E_i \cap B)}{P(B)} = \frac{P(B | E_i) P(E_i)}{P(B \cap E_1) + \dots + P(B \cap E_k)}$$

$$= \frac{P(B | E_i) P(E_i)}{P(B | E_1) P(E_1) + \dots + P(B | E_k) P(E_k)}$$

EX 15.

Nissan

$$P(S) = 0.5$$

$$P(M) = 0.3$$

$$P(P) = 0.2$$

D = "event of having problem in ignition system"

$$P(D | S) = 0.12$$

$$P(D | M) = 0.15$$

$$P(D | P) = 0.25$$

$$P(D) = P(D \cap S) + P(D \cap M) + P(D \cap P)$$

$$P(D) = P(D | S) P(S) + P(D | M) P(M) + P(D | P) P(P)$$

$$= (0.12)(0.5) + (0.15)(0.3) + (0.25)(0.2) = 0.155$$

(2)

$$P(P | D) = \frac{P(P \cap D)}{P(D)}$$

$$= \frac{P(D | P) P(P)}{P(D)} = \frac{(0.25)(0.2)}{0.155} = 0.3226$$