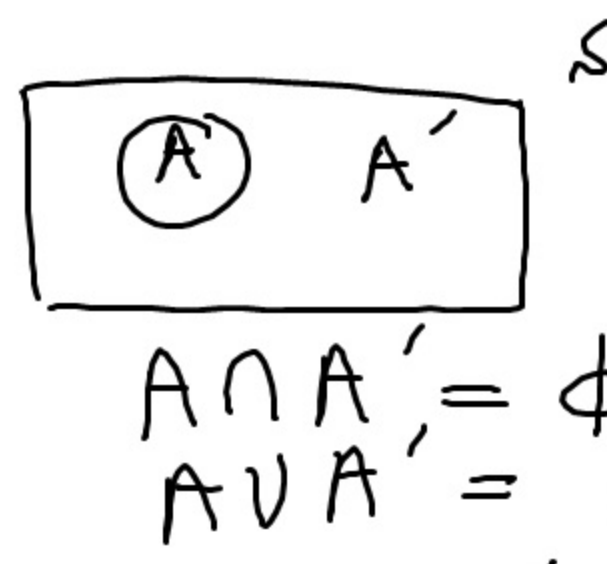
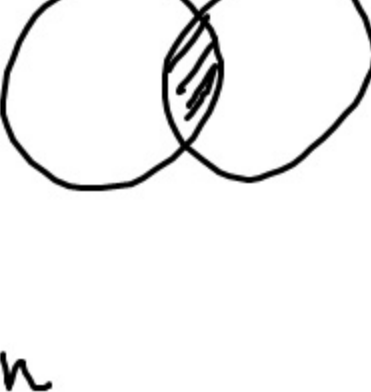


Review

$A \cup B$



$A \cap B$



$$A \cap A' = \emptyset$$

$$A \cup A' = S$$

$E_1, \dots, E_n$

$$(E_1 \cup \dots \cup E_n)' = E_1' \cap \dots \cap E_n'$$

$$(E_1 \cap \dots \cap E_n)' = E_1' \cup \dots \cup E_n'$$

$E_1, \dots, E_n$ are ME $E_i \cap E_j = \emptyset$ $i \neq j$
are exhaustive $E_1 \cup \dots \cup E_n = S$

Partition on S Both ME and Exhaustive

$$|E_1 \cup \dots \cup E_n| = |S|$$

Counting:

Ordered samples: $472 \quad 247$

with replacement: 724

$$\frac{n}{n} \cdot \frac{n}{n} \cdot \dots \cdot \frac{n}{n} = n^r$$

without replacement: Permutation

$$\text{order matters: } \frac{n!}{(n-r)!} = nPr$$

$$\frac{n!}{(n-r)!} = nPr$$

$$nPr = \frac{n!}{(n-r)!}$$

unordered samples

(without replacement)

$$\text{Combination } nCr = \frac{n!}{r!(n-r)!}$$

$\leftrightarrow$ Binomial Coefficient

n chooser

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

② we take an unordered sample of size r (from a sample space n) and then we can rearrange them $r!$ ways

$$nPr = nCr \cdot r!$$

$$nCr = \frac{nPr}{r!} = \frac{n!}{(n-r)! r!}$$

Ex 6. change 3 chips.

5 different types of chips.

$\rightarrow$ you can distinguish between different ordered samples.

$$\frac{5}{5} \cdot \frac{4}{4} \cdot \frac{3}{3} = 60 \quad 5P_3 = \frac{5!}{2!} = 60$$

Ex 7. 50 articles

50 $\rightarrow$ 47

$\rightarrow$ 3 defective.

we select 5.

a) select 5 out of 50

$$50C_5 = \frac{50!}{5!45!} = \frac{50 \times 49 \times 48 \times 47 \times 46}{4 \times 3 \times 2 \times 1 \times 1} = 2118760$$

b) 5 articles exactly 1 defective.

50 $\rightarrow$ 47 $\rightarrow$ choose 4 $\rightarrow \binom{47}{4}$ $\rightarrow$ stage 1

50 $\rightarrow$ 3 $\rightarrow$ choose 1 $\rightarrow \binom{3}{1}$ $\rightarrow$ stage 2.

$$\binom{3}{1} \binom{47}{4} = \frac{3!}{1!2!} \cdot \frac{47!}{4!43!} = 535095$$

Ex 8. 5 Candidates.

manager assistant manager secretary.

$$\frac{5}{M} \cdot \frac{4}{AM} \cdot \frac{3}{S} = 60$$

$$5P_3 = \frac{5!}{2!} = 60$$

Ex 9. 3 types of holes $\rightarrow$ small $\rightarrow$ Medium $\rightarrow$ Large

we want 2 small, 2 medium and 3 Large.

SSMMMLLL

SSMMMLLL

$$\binom{7}{2} \binom{5}{2} \binom{3}{3} = \frac{7!}{2!5!} \cdot \frac{5!}{2!3!} \cdot \frac{3!}{3!0!} = \frac{7!}{2!2!3!}$$

In general: the number of distinct permutation of n things of which n_1 are of one kind n_2 of the second kind $\dots$ n_k of the k th kind is

$$\frac{n!}{n_1! \dots n_k!}$$

Ex 10. 7 students 4 triple 2 double



$$\frac{7!}{3!2!2!} = 210$$

③ Interpretation of Probability

$P(A)$: A is an event, $P(A)$ is the probability that A occurs.

$$0 \leq P(A) \leq 1$$

attach a number between 0 and 1.

close to zero less likely to happen

close to 1, is more likely to happen.

$$P(S) = 1 \quad P(\emptyset) = 0$$

Probability

subjective.

$\rightarrow$ Equally likely model

Roll a die $S = \{1, 2, 3, 4, 5, 6\}$

$A =$ "event to see 3" $P(A) = \frac{1}{6}$

$B =$ "event to see odd" $\{1, 3, 5\}$ $P(B) = \frac{3}{6} = \frac{1}{2}$

$$P(E) = \frac{N(E)}{N(S)} = \frac{\# \text{ favourable outcomes}}{\# \text{ total possible outcomes}}$$

Sample space should be finite.

$\rightarrow$ Frequency Approach: repeat the same experiment

$$P(E) = \lim_{n \rightarrow \infty} \frac{f_n(E)}{n}$$

Where $f_n(E)$ is the number of times that event E occurs in n trials

Axioms of Probability:

E event S sample space

$$P(S) = 1 \quad P(\emptyset) = 0$$

① $P(E) \geq 0$ positivity

② $P(S) = 1$ certainty

③ $E_1, E_2, \dots$ Additivity

are ME ($E_i \cap E_j = \emptyset$, $i \neq j$)

$$P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$$

$$P(E_1 \cup E_2 \cup E_3 \dots) = P(E_1) + P(E_2) + \dots$$

$$S = A \cup A'$$

$$P(S) = P(A \cup A') = P(A) + P(A')$$

"

$$P(A') = 1 - P(A)$$

$$E_1 \subseteq E_2$$

Roll a die $E_1 = \{2\}$ $E_2 = \{2, 4, 6\}$

$$P(E_1) \leq P(E_2)$$

A, B

are ME

$$A = (A \cap B) \cup (A \cap B')$$



$$P(A) = P(A \cap B) + P(A \cap B')$$

A and B not ME

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$- P(A \cap B) - P(A \cap C) - P(B \cap C)$$

$$+ P(A \cap B \cap C)$$

DeMorgan Law

$$P(A') = 1 - P(A)$$

$$P(E_1 \cup \dots \cup E_n) = 1 - P(E_1' \cap \dots \cap E_n')$$

$$= 1 - P(E_1' \cap \dots \cap E_n')$$

$$P(E_1 \cap \dots \cap E_n) = 1 - P(E_1 \cup \dots \cup E_n) = 1 - P(E_1' \cap \dots \cap E_n')$$