MAT 2377 - Lecture 1 - Jan 9th 2018

Probability

A **random experiment**: is an experiment for which the outcome cannot be predicted with certainty.

Sample space: is a set of all possible outcomes.

- Ex: roll a die $S = \{1,2,3,4,5,6\}$ Discrete, finite.

  Flip a coin $S = \{\text{Head}, \text{Tail}\}$

- # of siblings you have $S = \{0,1,2,3,4,\dots\}$ infinite, discrete.

- amount of time waiting to catch the next bus

  1.334 $S = [0, \infty)$ Continuous, infinite

Event: is a collection of outcomes.

Roll a die

A = "The event that the outcome is an odd number" $A = \{1,3,5\}$

B = "The event that the outcome is prime" $B = \{2,3,5\}$

$A \cup B = \{1,2,3,5\}$ OR: at least in one of them

$A \cap B = \{3,5\}$ AND: in both of them

A^c (A'): complement event: is the set of all outcomes in S that are not in A. $A' = \{2,4,6\}$

$A \cap A' = \emptyset$ [] null set — Impossible event

$A \cup A' = S$ — certain event

n events $E_1, \dots, E_n$

$E_1 \cup E_2 \dots \cup E_n$: at least one of the events ($E_1 - E_n$) occurs

$E_1 \cap E_2 \dots \cap E_n$: All the events must occur @ the same time.

DeMorgan Law's

1. $(E_1 \cup \dots \cup E_n)' = (\text{At least one occurs})'$
   $= (\text{None of the events occur})$
   $= E_1' \cap E_2' \cap \dots \cap E_n'$ and
2. $(E_1 \cap E_2 \dots \cap E_n)' = (\text{All the events occur})'$
   $= (\text{At least one of the events does not occur})$
   $= E_1' \cup E_2' \cup \dots \cup E_n'$ OR

Mutually Exclusive events (ME)

If A and B has no outcomes in common, we say they are ME events — $A \cap B = \emptyset$

If one occurs, the other can't occur.

Rolling a die: $A = \{1,3,5\}$ $B = \{2\}$

A and B ME

$A \cap B = \emptyset$

$E_1, \dots, E_n$ events are ME

$E_i \cap E_j = \emptyset$ $i \neq j$ for i, j

Exhaustive events:

$E_1, \dots, E_n$ are said to be exhaustive if $E_1 \cup \dots \cup E_n = S$

Roll a die $A = \{1,3,5\}$ $B = \{1,2,4,6\}$

$A \cup B = \{1\}$ $A \cup B = \{1,2,3,4,5,6\} = S$ exhaustive

Partition: $E_1, \dots, E_n$ are ME + Exhaustive events, we have a partition on Sample space.

$E_1 \cup \dots \cup E_n = S$

$E_i \cap E_j = \emptyset$ $\forall i, j$ $(i \neq j)$.

Ex: A and A' (for any event A), is always a partition of S.

even if $A = \emptyset$ $A' = S$

Counting techniques.

1. How many ways roll a die then draw a card

   First stage: 6 ways to roll a die

   Stage 2: 52 different ways for the card.

   $6 \times 52 = 312$ (First stage × second stage)

2. Draw two tickets (1 – 100)

   first right hand then left hand.

   without replacement

   Stage 1: 100 ways to pick the first card

   Stage 2: 99 " " second card.

   $100 \times 99 = 9900$.

   In general: if we have K-stage procedure:

   n_1 ways for the first stage
   n_2 " " second "
   :
   n_k " " Kth "

   There are $n_1 n_2 \dots n_k$ ways. — multiply

Ordered Samples: N numbers: Draw a sample size of r

With replacement: $\underbrace{N \cdot N \cdots N}_{r \text{ times}} = N^r$

Without replacement: $N \cdot (N-1) \cdot \dots \cdot (N-r+1)$ ($N-(r-1)$ stages)

Permutation: nPr

When the order matters!

$0! = 1$

$n! = n \times (n-1) \dots 1$

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$1! = 1$

$nPr = \frac{n!}{(n-r)!} = n(n-1)\dots(n-r+1)$

$5P_3 = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 60$ — $5 \cdot 4 \cdot 3$

Ex3. 6 balls in order without replacement 1–49.

$44 \cdot 43 \cdot 42 \cdot 41 \cdot 40 \cdot 39 = 5174400$

$44P_6 = \frac{44!}{(44-6)!} = \frac{44!}{38!}$

Ex4. $\{0,1,\dots,9\}$ 6 digits

PIN Code you can create.

With replacement: $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^6$

Ex5. $\{a,b,c,d\}$ set of all permutations of size 2.

ab	ba
ac	ca
ad	da
bc	cb
bd	db
cd	dc

$4P_2 = \frac{4!}{2!} = \frac{4 \times 3 \times 2 \times 1}{2 \times 1} = 12$

Order matters & ab is different than ba

If the order does not matter {ab, ac, ad, bc, bd, cd} Combination = 6.