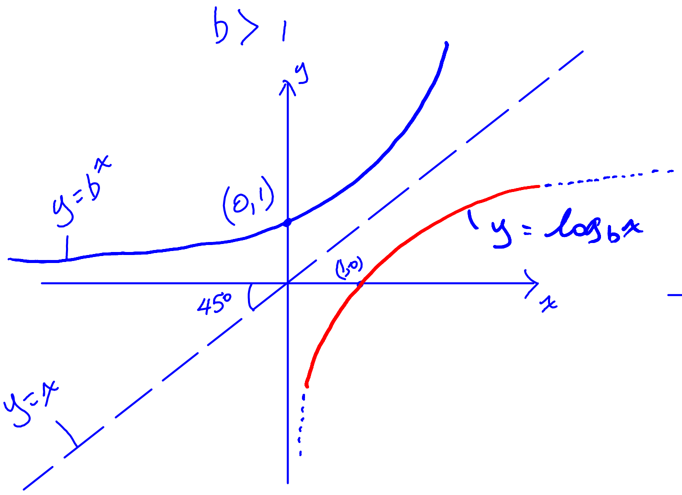
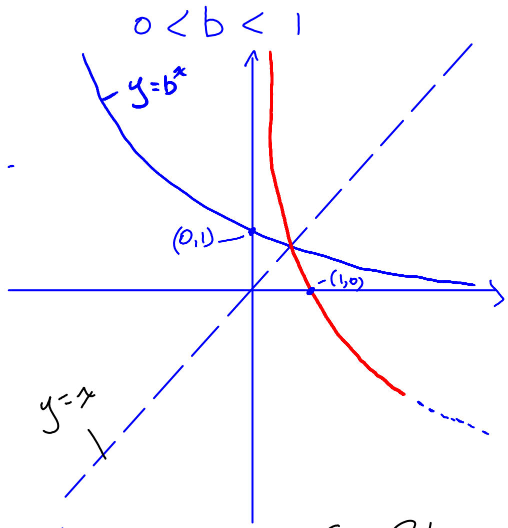
Graphical representation of logarithmic functions





$\text{domain}(\log_b x) = \text{ran}(b^x) = (0,\infty) = \{x \in \mathbb{R} \mid x > 0\}$

$\text{domain}(b^x) = \text{ran}(\log_b x) = \mathbb{R}$

NOTE: for $b>1 \Rightarrow$ "log" is increasing ... but!

ex. $b=10 \Rightarrow$

x	$\log x$
1	0
10	1
100	2
1000	3
10 000	4
⋮	

Differentiation Rules for exponential and logarithmic functions

$f(x)$	$f'(x)$
b^x	$(\ln b)\, b^x \longrightarrow$ ex. $f(x)=3^x \Rightarrow f'(x) = (\ln 3)\, 3^x$
if $b=e \Rightarrow f(x)=e^x$	$(\ln e)\, e^x = e^x \longrightarrow$ NOTE: SLOPE OF TANGENT $= f(x)$
$b^{g(x)}$	$f'(g(x))\cdot g'(x) = (\ln b)\, b^{g(x)}\cdot g'(x) \longrightarrow$ ex. $f(x) = 4^{(x^2+3x)}$ $\Rightarrow f'(x) = \underbrace{(\ln 4)}_{(\ln b)}\, \underbrace{4^{(x^2+3x)}}_{b^{g(x)}} \cdot \underbrace{(2x+3)}_{g'(x)}$
if $b=e \Rightarrow e^{g(x)}$	$e^{g(x)}\cdot g'(x)$

Differentiation for the logarithmic f^{ns}.

$f(x)$	$f'(x)$
$\log_b x$	$\left(\frac{1}{\ln b}\right)\frac{1}{x}$
if $b=e \Rightarrow \ln x$	$\left(\frac{1}{\ln e}\right)\frac{1}{x} = 1/x$
$\log_b(g(x))$	$f'(g(x))\cdot g'(x) = \left(\frac{1}{\ln b}\right)\frac{1}{g(x)}\cdot g'(x) = \frac{g'(x)}{(\ln b)\,g(x)}$
if $b=e \Rightarrow \ln(g(x))$	

$$\Downarrow$$

$$f'(x) = \frac{1}{g(x)}\cdot g'(x) = \frac{g'(x)}{g(x)}$$

$\hookrightarrow f(x) = \log_5(x^2+2^x)$

$$\Rightarrow f'(x) = \left(\frac{1}{\ln 5}\right)\cdot\frac{1}{(x^2+2^x)}\cdot\left(2x+(\ln 2)2^x\right)$$

$$f'(x) = \frac{2x+(\ln 2)2^x}{(\ln 5)(x^2+2^x)}$$

ARITHMETIC and GEOMETRIC SERIES

Recall the arithmetic and geometric sequences (n terms):

Arithmetic : $\underbrace{\underset{\text{FIRST TERM}}{A}, A+d, A+2d, A+3d, \dots, \underset{\text{LAST TERM}}{A+(n-1)d}}_{n \text{ term arithmetic sequence}}$ (COMMON DIFF. "d")

Geometric : $\underbrace{\underset{\text{FIRST TERM}}{A}, AR, AR^2, AR^3, \dots, \underset{\text{LAST TERM}}{AR^{(n-1)}}}_{n \text{ term geometric sequence}}$ (COMMON RATIO "R")

Def. A series is the sum of all the terms in a sequence. It is denoted S_n. It can be finite or infinite.

Formulas for the values of arithmetic and geometric series:

ARITHMETIC : $\boxed{S_n = nA + \frac{n(n-1)}{2}d}$

PROOF: $S_n = A + (A+d) + (A+2d) + \dots + (A+(n-1)d)$

also: $S_n = \underbrace{A + A + \dots + A}_{n \text{ times}} + (d + 2d + 3d + \dots + (n-1)d)$

$$S_n = nA + \underbrace{(1+2+3+4+\dots+(n-1))}_{\substack{\text{SUM OF FIRST} \\ (n-1) \text{ natural} \\ \text{numbers}}} d \qquad \frac{(n-1)n}{2}$$

$$\therefore \quad \boxed{S_n = nA + \frac{n(n-1)}{2} d}$$

Geometric SERIES: $S_n = A \cdot \left(\frac{R^n - 1}{R - 1} \right)$

ex. Consider: $1 + 3 + 9 + 27 + 81 = \boxed{121}$ $\begin{cases} A = 1 \\ n = 5 \\ R = 3 \end{cases}$

Also: $S_n = A\left(\frac{R^n - 1}{R - 1}\right) = (1)\left(\frac{3^5 - 1}{3 - 1}\right) = \frac{243-1}{2} = \frac{242}{2} = 121$

PROOF: $S_n = \underbrace{A + AR + AR^2 + \dots + AR^{(n-1)}}_{n \text{ terms}}$ — ①

$\times$① by R: $RS_n = AR + AR^2 + AR^3 + \dots + AR^n$ — ②

Now ②-①: $RS_n - S_n = AR^n - A$

$S_n(R-1) = A(R^n - 1) \Rightarrow \boxed{S_n = A \frac{(R^n - 1)}{R - 1}}$

Special case, where R is a fraction $|R| < 1$

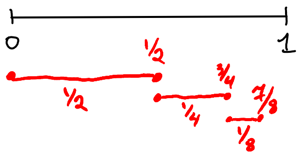
$-1 < R < 1$

For this case: $R^n \to 0$ as $n \to \infty$

$\therefore$ For an infinite series, where $|R| < 1$, the sum converges to the following value:

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[A\,\frac{R^n-1}{R-1}\right] = A\cdot\frac{-1}{R-1} = \boxed{\frac{A}{1-R}}$$

ex. Consider the following infinite series:



$$S = \frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\dots = 1$$

→ CONFIRM with $\dfrac{A}{1-R}$

$A = 1/2$, $R = 1/2$ } $S = \dfrac{A}{1-R} = \dfrac{1/2}{1-1/2} = 1$

PART 2

Some ECON APPLICATIONS

Section 10.4

Simple and Compound Interest

See illustrative examples in tables 10.2 and 10.3 of this section. Compare these examples with the analysis below.

Premise behind interest

- Start with an initial amount of money, called the initial capital (denoted A)
- Observe the amount increase over time, as interest is paid out at regular intervals. This amount that changes over time is the cumulative amount C. A.
- Interest is computed according to 2 different schemes:

rA ⇐ → SIMPLE INTEREST } THE CURRENCY AMOUNT is some percentage r of the initial capital A. (always!)

$r(C.A)$ ⇐ → COMPOUND INTEREST } The currency amount of interest is some percentage r of the cumulative amount C. A.