# CONCORDIA UNIVERSITY
## Department of Mathematics & Statistics

| **Course** | **Number** | **Sections** |
|---|---|---|
| Mathematics | 205 | All |
| **Examination** | **Date** | **Pages** |
| Final | December 2017 | 2 |
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| **Special Instructions:** | Only approved calculators are allowed. **Show all your work for full marks.** | |

*MARKS*

[10] **1.** **a.** Sketch the graph of the integrand of $\int_{-1}^{2} (1 - |x|)\, dx$ and evaluate the integral in terms of area (do **not** antidifferentiate).

**b.** Use the Fundamental Theorem of Calculus to calculate the derivative of $F(x) = \int_{2^x}^{3^x} \sqrt[3]{t}\, dt$, and determine whether $F$ is increasing or decreasing at $x = 9$.

[10] **2.** Find the following indefinite integrals:

(**a**) $\displaystyle\int x\,(\ln x)^2\, dx$ (**b**) $\displaystyle\int \frac{6x+7}{(x+2)^2}\, dx$

[6] **3.** Find $F(x)$ such that $F'(x) = \sqrt{\dfrac{x^4}{x^3+1}}$ and $F(0) = 1$.

[18] **4.** Evaluate the following definite integrals (give the **exact** answers, do not approximate):

(**a**) $\displaystyle\int_0^3 \frac{dx}{\sqrt{16+x^2}}$ (**b**) $\displaystyle\int_1^{e^{\pi/4}} \frac{4}{x\,(1+(\ln x)^2)}\, dx$ (**c**) $\displaystyle\int_{-\pi/4}^{\pi/4} \tan^4 x\, dx$

[8] **5.** Evaluate the given improper integral or show that it diverges:

(**a**) $\displaystyle\int_{-\infty}^{0} xe^x\, dx$ (**b**) $\displaystyle\int_0^1 \frac{dx}{x - \sin x}$

[16] **6.** **a.** Sketch the curves $x = y^2$ and $x = y^3$, and find the area enclosed.

**b.** Find the volume of the solid obtained by rotating the region bounded by the curve $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

**c.** Show that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ for any function $f(x)$ continuous on $[0, a]$.

[6] **7.** Find the limit of the sequence $\{a_n\}$ as $n \to \infty$ or prove that it does not exist:

**(a)** $a_n = \dfrac{e^n}{n!}$ **(b)** $a_n = (-1)^n \dfrac{\sin(n)}{n}$

[12] **8.** Determine whether the series is divergent or convergent, and if convergent, whether absolutely or conditionally :

**(a)** $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{n+1}{n^2\sqrt{n+2}}$ **(b)** $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n \ln(n)}{\sqrt{n}}$ **(c)** $\displaystyle\sum_{n=0}^{\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$

[6] **9.** Find **(a)** the radius of convergence, and **(b)** the interval convergence of the series $\displaystyle\sum_{n=0}^{\infty}\frac{(3x-2)^n}{5^n}$ .

[8] **10.** **(a)** Derive the Maclaurin series of $f(x) = \dfrac{1}{1+x^2}$ .

(HINT: start with the series for $\dfrac{1}{1-z}$ where $z = -x^2$).

**(b)** Use integrability of the power series derived in (a) to find the power series for $\arctan(x)$.

[5] **Bonus question.** Use the formula given above in **6 c** to evaluate $\displaystyle\int_0^{\pi/2}\frac{\sin x}{\sin x + \cos x}\,dx$.