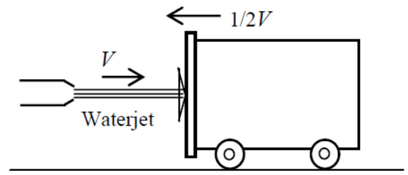
CVG 2516 – Mécanique élémentaires des Fluides

Devoir 5 - Solutions

1.

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V} \rightarrow -F_R = -\dot{m}_i V_i \rightarrow F_R = \dot{m}_i V_i$$



Stationary plate: $(V_i = V$ and $\dot{m}_i = \rho A V_i = \rho A V) \rightarrow F_R = \rho A V^2 = F$

Moving plate: $(V_i = 1.5V$ and $\dot{m}_i = \rho A V_i = \rho A(1.5V))$

$$\rightarrow F_R = \rho A(1.5V)^2 = 2.25 \rho A V^2 = 2.25F$$

2. Point 1 = Surface en amount de la porte et Point 2 = Surface en aval de la porte.

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + y_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + y_2 \rightarrow V_2^2 - V_1^2 = 2g(y_1 - y_2)$$

$$\dot{V}_1 = \dot{V}_2 = \dot{V} \rightarrow A_1 V_1 = A_2 V_2 = \dot{V} \rightarrow V_1 = \frac{\dot{V}}{A_1} = \frac{\dot{V}}{w y_1} \quad \text{and} \quad V_2 = \frac{\dot{V}}{A_2} = \frac{\dot{V}}{w y_2}$$

$$\left(\frac{\dot{V}}{w y_2}\right)^2 - \left(\frac{\dot{V}}{w y_1}\right)^2 = 2g(y_1 - y_2) \rightarrow \dot{V} = w\sqrt{\frac{2g(y_1 - y_2)}{1/y_2^2 - 1/y_1^2}} \rightarrow \dot{V} = w y_2 \sqrt{\frac{2g(y_1 - y_2)}{1 - y_2^2/y_1^2}}$$

$$V_1 = \frac{y_2}{y_1}\sqrt{\frac{2g(y_1 - y_2)}{1 - y_2^2/y_1^2}} \quad \text{and} \quad V_2 = \sqrt{\frac{2g(y_1 - y_2)}{1 - y_2^2/y_1^2}}$$

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$$

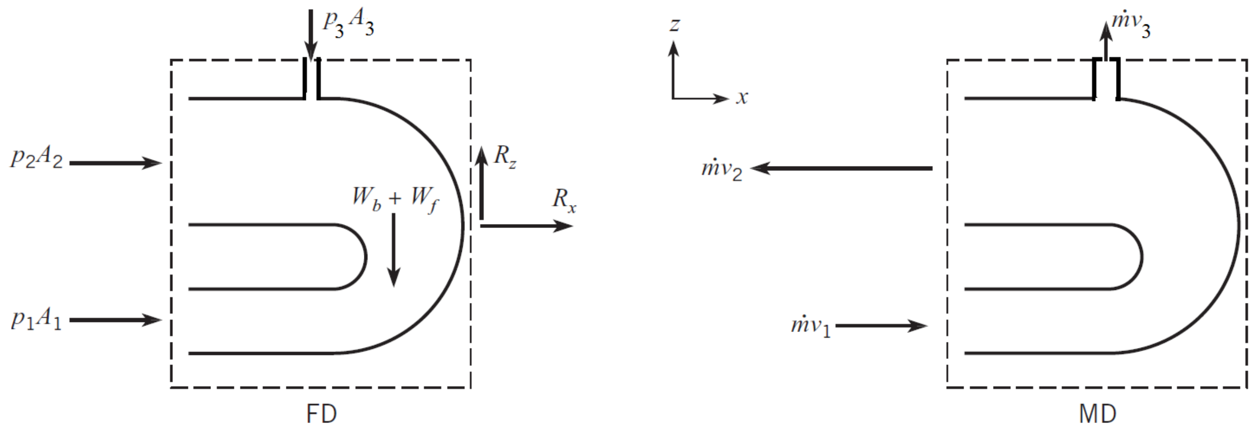
$$-F_{Rx} + P_1 A_1 - P_2 A_2 = \dot{m} V_2 - \dot{m} V_1 \rightarrow -F_{Rx} + \left(\rho g \frac{y_1}{2}\right)(w y_1) - \left(\rho g \frac{y_2}{2}\right)(w y_2) = \dot{m}(V_2 - V_1)$$

$$F_{Rx} = \dot{m}(V_1 - V_2) + \frac{w}{2}\rho g(y_1^2 - y_2^2)$$

Comme y1 >> y2, alors:

$$\dot{V} = y_2 w \sqrt{2 g y_1} \quad \text{or} \quad V_2 = \sqrt{2 g y_1}$$

3.



$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho \, dV + \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix}$$

$$\sum F_z = \frac{d}{dt}\int_{cv} v_z \rho \, dV + \sum_{cs} \dot{m}_o v_{oz} - \sum_{cs} \dot{m}_i v_{iz}$$

En utilisant FD:

$$\sum F_x = p_1 A_1 + p_2 A_2 + R_x$$

$$\sum F_z = R_z - p_3 A_3$$

$P_1 = 200\ kPa\ (abs);\ P_2 = 150\ kPa\ (abs);\ P_3 = 100\ kPa\ (abs)$

En utilisant MD:

Sortant:

x-direction: $$\sum_{cs} \dot{m}_o v_{ox} = \dot{m}(-v_2)$$

z- direction: $\sum \dot{m}_o v_{oz} = \dot{m} v_3$

Entrant:

x-direction: $$\sum_{cs} \dot{m}_i v_{ix} = \dot{m} v_1$$

z- direction: $\sum \dot{m}_o v_{oz} = 0$

$$V = \frac{Q}{A}$$

$$A = \frac{\pi D^2}{4}$$

$$\dot{m} = \rho Q$$

$$V = \frac{\dot{m}}{\rho A}$$

Pour chaque section:

$V_1 = 15.29\ m/s$; $V_2 = 2.8\ m/s$; $V_3 = 1.13\ m/s$.

On combine le tout :

X- : $R_x = -P_1 A_1 - P_2 A_2 - \dot{m}_2 V_2 - \dot{m}_1 V_1 = -2090.3\ N$

Z-: $R_z = P_3 A_3 + \dot{m}_3 v_3 = 79.69\ N$