SOLUTION FINAL APRIL 2015

Q1

a/ $4\sqrt{12} + 5\sqrt{27} - \sqrt{75} = 8\sqrt{3} + 15\sqrt{3} - 5\sqrt{3} = 18\sqrt{3}$

b/ $\frac{1}{3}\log_3 27 - \log_3(3^3 - 18) = \log_3 27^{1/3} - \log_3 9 = \log_3 (3^3)^{1/3} - \log_3 3^2 = 1 - 2 = -1$

Q2

a/ $\dfrac{5}{2\sqrt{3}} = \dfrac{5\sqrt{3}}{2\sqrt{3}\cdot\sqrt{3}} = \dfrac{5\sqrt{3}}{6}$

b/ $\dfrac{2-\sqrt{5}}{2+3\sqrt{5}} = \dfrac{2-\sqrt{5}}{2+3\sqrt{5}} \cdot \dfrac{2-3\sqrt{5}}{2-3\sqrt{5}} = \dfrac{(2-\sqrt{5})(2-3\sqrt{5})}{4-45} = -\dfrac{(2-\sqrt{5})(2-3\sqrt{5})}{41}$

Q3

a/ $5x(x^3-5x^2) - x^2(x^2-7x-5)$

$= 5x^4 - 25x^3 - x^4 + 7x^3 + 5x^2$

$= 4x^4 - 18x^3 + 5x^2$

b/ $\dfrac{x^3-8}{x^3-2x^2} = \dfrac{(x-2)(x^2+2x+4)}{x^2(x-2)} = \dfrac{x^2+2x+4}{x^2}$

Q4

a/ $4x^2 - 16x + 15 = (2x-3)(2x-5)$

b/ $1 - 8x^2 - 9x^4 = 1 - 9x^2 + x^2 - 9x^4 = (1+x^2) - 9x^2(1+x^2) = (1+x^2)(1-9x^2) = (1+x^2)(1-3x)(1+3x)$

Q5

$$\frac{4x}{x^2-4} - \frac{2}{x^2+x-6} = \frac{4x}{(x-2)(x+2)} - \frac{2}{(x+3)(x-2)} = \frac{4x(x+3)}{(x-2)(x+2)(x+3)} - \frac{2(x+2)}{(x-2)(x+2)(x+3)}$$

$$= \frac{4x^2+12x-2x-4}{(x-2)(x+2)(x+3)} = \frac{4x^2+10x-4}{(x-2)(x+2)(x+3)}$$

Q6

a/ $\dfrac{2x}{x^2-4} = \dfrac{4}{x^2-4} - \dfrac{3}{x+2}$

$\Leftrightarrow 2x = 4 - 3(x-2)$

$\Leftrightarrow 2x = 4 - 3x + 6$

$\Leftrightarrow 5x = 10$

$\Leftrightarrow x = 2$ (rejected)

b/ $\log_3(3x-1) = 2$

$\Leftrightarrow 3x - 1 = 3^2 = 9$

$\Leftrightarrow 3x = 10$

$\Leftrightarrow x = \frac{10}{3}$ (accepted)

c/ $3^{x^3} = 9^x$

$\Leftrightarrow 3^{x^3} = 3^{2x}$

$\Leftrightarrow x^3 = 2x$

$\Leftrightarrow x^3 - 2x = 0$

$\Leftrightarrow x(x^2-2) = 0$

$\Leftrightarrow x(x-\sqrt{2})(x+\sqrt{2}) = 0$

$\Rightarrow x = 0;\ x = \sqrt{2};\ x = -\sqrt{2}$

Q7

a/ $-1 \leq \dfrac{3-5x}{2} \leq 9$

$\Leftrightarrow -2 \leq 3 - 5x \leq 18$

$\Leftrightarrow -5 \leq -5x \leq 15$

$\Leftrightarrow 1 \geq x \geq -3$

b/ $\left|\dfrac{2x+3}{3} - \dfrac{1}{2}\right| < 1$

$\Leftrightarrow \left|\dfrac{2(2x+3)-3}{6}\right| < 1$

$\Leftrightarrow -1 < \dfrac{4x+3}{6} < 1$

$\Leftrightarrow -6 < 4x + 3 < 6$

$\Leftrightarrow -9 < 4x < 3 \Rightarrow \boxed{\frac{-9}{4} < x < \frac{3}{4}}$