NAME: ____________________________________________ STUDENT NUMBER: ____________________

SECTION: A__, B__,

**C A R L E T O N   U N I V E R S I T Y**

**MAAE 2300 Mid-Term Test**
**October 2017**

DURATION: **24 Hours**

No. of Students: 275

Department: Mechanical and Aerospace Engineering

Course Number: **MAAE 2300 Fluid Mechanics I**

Instructors: John Gaydos (Section A), Thor Jodoin (Section B)

**Please indicate your Section along with your Name and Student Number.**

**Show your final answers on separate pages and hand-in this exam paper and your additional pages.**

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**MULTIPLE CHOICE ANSWERS**

**For Questions 1-9, a brief explanation is required.**
**For Questions 10-18, you must provide a complete solution.**

| SECTION | Questions | Value of Each Question | Total |
|---|---|---|---|
| Section 1: Multiple Choice Questions (Brief explanations required) | 1-9 | 2 marks | 18 marks |
| Section 2: Short Answer Questions (Full solutions required) | 10-17 | 4 marks | 32 marks |
| Section 3: Long Answer Question (Full solutions required) | 18 | 10 marks | 10 marks |
| | | **TOTAL** | **60 marks** |

Detailed breakdown of question marks:

| **Question No.:** | **Marks** |
|---|---|
| 1-9 | 9 x 2 |
| 10-17 | 8 x 4 |
| 18 | 1 x10 |
| **TOTAL MARKS:** | **60** |

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## Section 1: Multiple Choice Questions (Brief calculation required).

***Please briefly explain your answers on separate pages.***

**Question 1 (Automatic 2 marks awarded, due to error in original soultion)**

Calculate the force needed to provide an initial upward acceleration of 35 $m/s^2$ to a 0.8 kg rocket.

(a) 7.85 N
(b) 20.15 N
(c) 28.00 N
(d) 15.70 N
(e) 19.92 N

**Q1 Solution:**

Sum of forces in the vertical y direction:

$$\Sigma F_y = ma_y$$

$$F - mg = ma$$

$$F = ma + mg$$

$$F = m(a + g)$$

$$F = 0.8\,kg \times (35\ m/s^2 + 9.81\,m/s^2)$$

$$\therefore \boldsymbol{F} = \mathbf{35.85\ N}$$

**Question 2 (2 marks)**

When a fluid moves past a solid, stationary wall, the speed of the fluid changes with distance from the solid wall. However, the fluid adjacent to the wall is stationary. This condition is known as:

(a) The no shear stress condition,
(b) The stationary condition,
(c) The solid wall principle of flow,
(d) The no slip condition,
(e) The no slide condition.

**Question 3 (2 marks)**

A $0.75\text{m}\ x\ 2.5\text{m}$ flat plate is towed at $6\ \text{m/s}$ on a $2.54\ \text{mm}$ thick layer of SAE-30 oil at $15°\text{C}$ [ dynamic viscosity of $0.175\ \ (\text{N}\cdot\text{s})/\text{m}^2$ ] that separates it from a flat surface. The velocity distribution between the plate and the surface is assumed to be linear.

What force is required if the plate and surface are horizontal?

(a) 775.10 N
(b) 175.00 N
(c) 413.39 N
(d) 2362.2 N
(e) 250.00 N

**Q3 Solution:**
Using the equation:

$$\tau = \mu \cdot \frac{\delta u}{\delta y}$$

The velocity gradient is calculated to be:

$$\frac{du}{dy} = \frac{\Delta u}{\Delta y} = \frac{6\ m/s}{0.00254\ m} = 2362.20\ \frac{1}{s}$$

Therefore:

$$\tau = 0.175\ \frac{N \cdot s}{m^2} \cdot 2362.20\ \frac{1}{s} = 413.39\ \frac{N}{m^2}$$

Force is stress multiplied by area:

$$F = \tau \cdot A = 413.39\ \frac{N}{m^2} \cdot (0.75\ m \cdot 2.5\ m)$$

$$\therefore \boldsymbol{F = 775.10\ N}$$

## Question 4 (2 marks)

Assume the plate in **Question 3** is 0.032 inches thick and made out of 2024-T3 aluminum ( $SG_{Al} = 2.78$ ).

If the surface tension of the oil is 0.031 N/m, how much force is required to lift the plate?

(a) 201.5 N
(b) 41.59 N
(c) 58.13 N
(d) 41.79 N
(e) 166.8 N

**Q4 Solution:**
Determine mass of plate:

$$m = \rho \cdot V$$

$$m = \left(2.78 \cdot 1000\ \frac{kg}{m^3}\right) \cdot \left(0.032\ in \cdot 0.0254\ \frac{m}{in} \cdot 0.75\ m \cdot 2.5\ m\right)$$

$$m = \left(2780\ \frac{kg}{m^3}\right) \cdot (1.524 \cdot 10^{-3}\ m^3)$$

$$m = 4.24\ kg$$

Determine force:

$$\Sigma F_y = mg + 2\sigma(\ell + w)$$

$$\Sigma F_y = \left(4.24\ kg \cdot 9.81\ \frac{m}{s^2}\right) + \left[2 \cdot 0.031\ \frac{N}{m} \cdot (2.5\ m + 0.75\ m)\right]$$

$$\Sigma F_y = (41.59\ N) + (0.20\ N)$$

$$\therefore \boldsymbol{F = 41.79\ N}$$

## Question 5 (2 marks)

You are conducting an experiment in an enclosed test chamber near Lake Louise, Alberta (which is 1,661 m above sea level).

If one of the pressures in your experiment reads as a vacuum of 23 kPa, then what is the absolute pressure in your test chamber?

Assume the following sea level reference conditions for that day:

| | | | |
|---|---|---|---|
| Sea Level | $z_0 = 0$ m | Pressure | $p_0 = 101.27$ kPa |
| Temperature | $T_0 = -3$°C | Lapse rate | $\alpha = -6.5$ K/km |
| Gas Constant | $R = 287$ J/(kg · K) | | |

(a) 78.13 kPa
(b) 82.26 kPa
(c) 78.27 kPa
(d) 59.26 kPa
(e) 79.50 kPa

**Q5 Solution:**

Calculate atmospheric pressure at Lake Louise:

$$T_0 = -3\text{°C} = 270.15\ \text{K}$$

$$\frac{p}{p_0} = \left[\frac{T_0 + \alpha \cdot (z - z_0)}{T_0}\right]^{-\frac{g}{\alpha \cdot R}}$$

$$p = p_0 \left[\frac{T_0 + \alpha \cdot (z - z_0)}{T_0}\right]^{-\frac{g}{\alpha \cdot R}}$$

$$p = 101.27\left[\frac{270.15 - 0.0065 \cdot (1611 - 0)}{270.15}\right]^{\frac{9.81}{0.0065 \cdot 287}}$$

$$p = 101.27\,[0.96]^{5.26}$$

$$p = 101.27 \times 0.81$$

$$p = 82.26\ \text{kPa}$$

Calculate absolute pressure in test chamber:

$$p_{absolute} = p_{atmosphere} + p_{gage}$$

$$p_{absolute} = 82.26\ \text{kPa} + (-23\ \text{kPa})$$

$$\boldsymbol{p_{absolute} = 59.26\ \text{kPa}}$$

## Question 6 (2 marks)

Two meters of water is equivalent to how many millimeters of mercury?

( Assume $SG_{Hg} = 13.6$ )

(a) 231 mm
(b) 422 mm
(c) 147 mm
(d) 375 mm
(e) 211 mm

**Q6 Solution:**

$$p = \gamma \cdot h$$

$$p_{water} = p_{Hg}$$

$$\gamma_{water} \cdot h_{water} = \gamma_{Hg} \cdot h_{Hg}$$

$$\gamma_{water} \cdot h_{water} = \left(SG_{Hg} \cdot \gamma_{water}\right) \cdot h_{Hg}$$

$$h_{Hg} = \frac{h_{water}}{SG_{Hg}}$$

$$\boldsymbol{h_{Hg} = \frac{2\ m}{13.6} = 0.147\ m = 147\ mm}$$

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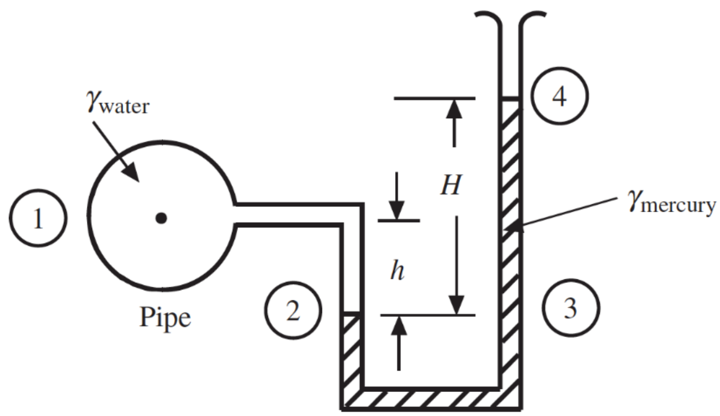
## Question 7 (2 marks)

What is the gauge pressure in the water pipe (shown in the figure below) if $h = 15$ cm and $H = 25$ cm?

( Assume $SG_{Hg} = 13.6$ )




(a) 22.8 kPa
(b) 27.3 kPa
(c) 31.9 kPa
(d) 39.1 kPa
(e) 28.2 kPa

**Q7 Solution:**

$$p_{water} + \gamma_{water} \cdot h = \gamma_{Hg} \cdot H$$

$$p_{water} = \gamma_{Hg} \cdot H - \gamma_{water} \cdot h$$

$$p_{water} = \gamma_{water} \cdot \left( SG_{Hg} \cdot H - h\right)$$

$$p_{water} = 9810 \, \frac{N}{m^3} \cdot ( 13.6 \cdot 0.25 \, m - 0.15 \, m)$$

$$p_{water} = 9810 \, \frac{N}{m^3} \cdot ( 3.4 \, m - 0.15 \, m)$$

$$p_{water} = 9810 \, \frac{N}{m^3} \cdot ( 3.25 \, m)$$

$$\boldsymbol{p_{water} = 31{,}883 \, kPa = 31.9 \, kPa}$$

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## Question 8 (Automatic 2 marks awarded, due to error in original soultion)

A test tube is placed in a rotating device that grabs it by the top end (i.e. the open end) and, as it spins, gradually positions the tube outward in a horizontal position. The test tube is 12 cm long, 4 cm in diameter, and completely filled with water. At steady state the rotating device is spinning the tube at 1000 rpm.

What is the gauge pressure at the bottom of the test tube?

(a) 677 kPa
(b) 723 kPa
(c) 658 kPa
(d) 697 kPa
(e) 767 kPa

**Q8 Solution:**

$$p_2 - p_1 = \frac{\rho \cdot \omega^2 \cdot (r_2 - r_1)^2}{2}$$

$$p_2 - 0 = \frac{1000\ \frac{kg}{m^3} \cdot \left(1000\ rpm \cdot \frac{2\pi\ rad}{60\ sec/min}\right)^2 \cdot (0.12\ m - 0)^2}{2}$$

$$p_2 = \frac{1000\ \frac{kg}{m^3} \cdot \left(1000\ rpm \cdot \frac{2\pi\ rad}{60\ sec/min}\right)^2 \cdot (0.12\ m)^2}{2}$$

$$p_2 = \frac{1000\ \frac{kg}{m^3} \cdot \left(104.72\ \frac{rad}{sec}\right)^2 \cdot (0.12\ m)^2}{2}$$

$$\boldsymbol{p_2 = 78{,}957\ Pa = 79.0\ kPa}$$

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**Question 9 (2 marks)**

A 6 $cm$ diameter horizontal stationary water jet, having a velocity of 40 $m/s$, strikes a vertical plate. If the plate is moving away from the jet at 20 m/s, then what force will the plate feel from the water jet?

(a) 1365 N
(b) 1270 N
(c) 1131 N
(d) 1080 N

**Q9 Solution:**

$$F = \dot{m}_r (v_{r1x} - v_{r2x})$$

$$F = \rho A_r (v_{r1x} - v_{r2x})$$

$$F = 1000 \frac{kg}{m^3} \cdot (\pi \cdot 0.03^2) \cdot \left(40 \frac{m}{s} - 20 \frac{m}{s}\right)$$

$$\boldsymbol{F = 1131\ N}$$

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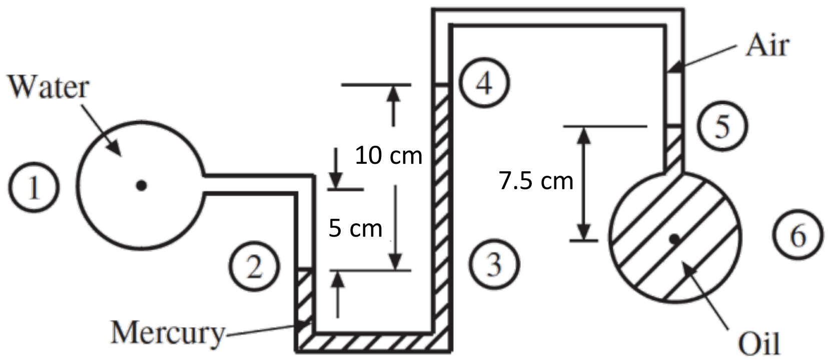
## Section 2: Short Answer Questions (Full Solutions Required)

***Please provide your answers on separate pages.***

### Question 10 (4 marks)

A manometer connects an oil pipeline and a water pipeline, as shown. What is the difference in pressure between the two pipelines, using the readings on the manometer?

Use $SG_{oil} = 0.86$ and $SG_{Hg} = 13.6$.




**Q10 Solution:**

The points of interest have been positioned on the manometer shown. The pressure at point 2 is equal to the pressure at point 3:

$$p_2 = p_3$$

$$p_{water} + (\gamma_{water} \cdot 0.05\ m) = p_4 + (\gamma_{Hg} \cdot 0.10\ m)$$

Observe that the heights must be in meters. The pressure at point 4 is essentially the same as at point 5 since the specific weight of air is negligible compared to that of oil, water or mercury. So:

$$p_4 = p_5$$

$$p_4 = p_{Oil} + (\gamma_{Oil} \cdot 0.075\ m)$$

Finally:

$$p_{water} + (\gamma_{water} \cdot 0.05\ m) = p_{Oil} + (\gamma_{Oil} \cdot 0.075\ m) + (\gamma_{Hg} \cdot 0.10\ m)$$

$$p_{water} - p_{Oil} = (\gamma_{Hg} \cdot 0.10\ m) + (\gamma_{Oil} \cdot 0.075\ m) - (\gamma_{water} \cdot 0.05\ m)$$

$$p_{water} - p_{Oil} = \gamma_{water} \cdot [(SG_{Hg} \cdot 0.10\ m) + (SG_{Oil} \cdot 0.075\ m) - 0.05\ m]$$

$$p_{water} - p_{Oil} = 9810\ \frac{N}{m^3} \cdot [(13.6 \cdot 0.10\ m) + (0.86 \cdot 0.075\ m) - 0.05\ m]$$

$$p_{water} - p_{Oil} = 9810\,\frac{N}{m^3}\cdot[(13.6\cdot 0.10\,m) + (0.86\cdot 0.075\,m) - 0.05\,m]$$

$$p_{water} - p_{Oil} = 9810\,\frac{N}{m^3}\cdot[(1.36\,m) + (0.0645\,m) - 0.05\,m]$$

$$p_{water} - p_{Oil} = 9810\,\frac{N}{m^3}\cdot 1.3745\,m$$

$$\boldsymbol{p_{water} - p_{Oil} = 13{,}484\,Pa = 13.48\,kPa}$$

## Question 11 (4 marks)

Explain why you either would or would not feel justified using Newton's law of viscosity in each these situations:

(a) Modeling the flow of caulk out of a caulking gun,
(b) Modeling the flow of water out of a squirt gun,
(c) Modeling the flow of hot gases out of an aircraft turbojet engine,
(d) Modeling the flow of magma out of a volcanic vent as lava.

We will assume that Newton's law is appropriate if the fluid is a gas, a liquid with a simple molecular structure and no additives, or a material that we have observed behaving like a Newtonian fluid under the conditions of interest.

**Q11 Answers:**

(a) The vinyl-based caulk used in most home improvement projects is a high molecular weight polymer and it does not flow out unless substantial pressure is applied with the plunger of the caulking gun. Thus, we conclude that caulk is not a Newtonian fluid.

(b) Water has a simple molecular structure and Newton's law of viscosity is applicable so water behaves as a Newtonian fluid.

(c) A turbojet exhaust stream is a mixture of hot gases; primarily air. There is no reason to think that the mixture will behave differently from any other gas because it exists at an elevated temperature. We conclude that the exhaust mixture is a Newtonian fluid and that Newton's law of viscosity is applicable.

(d) Magma is composed of molten rock and it is found in the Earth's crust. Lava is magma that reaches the surface of our planet through a volcanic vent and it flows as a complex mixture of solid-liquid-gaseous portions. Lava does not behave during its flow as a Newtonian fluid.

## Question 12 (4 marks)

In each of the following situations would it be justifiable to use the continuum hypothesis in your analysis of the fluid flow? Please explain for each situation what additional information you would like to obtain before making a final decision.

(a) Calculating the shear stress in a lubricant layer that is 1 nm thick,

(b) Modeling the flow of blood in the smallest capillaries of the body,

(c) Airflow over a military jet moving at Mach 1.3 when it is at an altitude of 2 km,

(d) It is possible to use semiconductor fabrication techniques to create gas flow valves that can be embedded directly into a tiny silicon wafer. Consider a device with characteristic dimension in the micrometer range and a gas pressure of about $10^{-4}$ atm. You are asked to predict the flow through these micro-valves.

To determine which of the situations listed can be adequately modeled with a continuum theory we will compare the length scale in the situation listed above to an appropriate molecular dimension such as the mean free path of a molecule in a gas.

**Q12 Answers:**

(a) Atoms typically have radii on the order of 0.1 nm so the thickness of the layer is similar in dimension. Therefore, it is not valid to use the continuum hypothesis to predict the shear stress in this problem.

(b) We should determine the diameter of the smallest capillaries and then compare it to the characteristic molecular dimension for blood (on the order of nanometers) to see if the continuum hypothesis is valid for this fluid system. In this case the continuum hypothesis does apply. However, we should also be aware that blood is a complex fluid made up of cells immersed within a plasma. The diameter of the smallest capillaries is approximately the same size as red blood cells. This creates an unusual flow as the red blood cells squeeze through them in single file.

(c) Obviously, the mean free path of gas molecules at ordinary pressure is much less than the characteristic length of a jet, so we should be justified using the continuum approximation in this case. Thus, by obtaining a relationship between gas pressure and mean free path, and a relationship between altitude and pressure, one could determine the range of altitudes for which the continuum hypothesis is useful.

(d) The combination of small dimensions and low gas pressure suggests that the continuum hypothesis is not valid.

## Question 13 (4 marks)

Briefly explain one modern or semi-modern method of measuring a fluid's dynamic viscosity.

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## Question 14 (4 marks)

Consider the following exponential equation for the relation between dynamic viscosity and temperature, where $A$ and $B$ are both constants for a given liquid:

$$\mu = A \cdot e^{\left(\frac{B}{T}\right)}$$

Calculate the temperature at which the dynamic shear viscosity of water is equal to 40% of its value at 20°C, given the experimental viscosity values: $\mu$( 20°C ) = 0.001 (N·s)/m$^2$ and $\mu$( 0°C ) = 0.0018 (N·s)/m$^2$.

**Q14 Answer:**

We use the experimental information given to obtain a value for the constant $B$ and to then use this value to determine the temperature at which the viscosity of water is 40% of its value at room temperature. The ratio of the viscosity of water at any two temperatures is given by

$$\frac{\mu_1}{\mu_2} = \frac{A \cdot exp(B/T_1)}{A \cdot exp(B/T_2)} = exp\left[B\left(\frac{1}{T_1} - \frac{1}{T_2}\right)\right]$$

Solving this expression for $B$ and substituting the values given in the problem statement (with the subscript '1' referring to the experimental value at 20°C and the subscript '2' to the value at 0°C) yields

$$B = \frac{ln(\mu_1/\mu_2)}{(1/T_1 - 1/T_2)} = \frac{ln\left[0.001(\frac{Ns}{m})/0.0018\ (\frac{Ns}{m})\right]}{1/293.16 - 1/273.16} = 2353\ K$$

Next, we solve the viscosity ratio equation for the temperature $T_2$ to obtain

$$T_3 = \left[\frac{1}{T_1} - \frac{ln(\mu_1/\mu_3)}{B}\right]^{-1}$$

Now, if subscript '1' continues to refer to state at 20°C and subscript '3' refers to the unknown temperature at which the viscosity is just 40% of the room temperature value ( i.e. $\mu_3/\mu_1 = 0.4$ ), then

$$\boldsymbol{T_2 = \left[\frac{1}{293.16} - \frac{ln(1/0.4)}{2353}\right]^{-1} = 330.94\ K = 57.78\ ℃}$$

## Question 15 (4 marks)

Consider the three-fluid flow situation illustrated in the sketch below. The oil has a specific gravity of 0.9.

(a) Suppose that water is flowing into the tank at exactly the same mass flow rate as that of the oil flowing out of the tank. In which direction, if at all, is air moving through the top vent?

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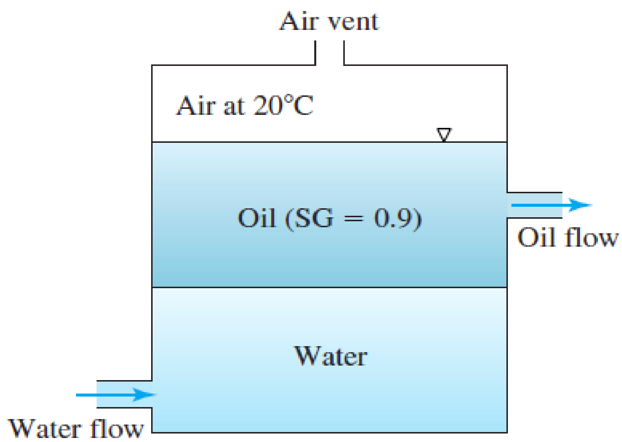
(b) Suppose that water is flowing into the tank at exactly the same volume flow rate as that of the oil flowing out of the tank. In which direction, if at all, is air moving through the top vent?

Briefly explain your answers, for each case.




**Q15 Answer:**

A control volume for this problem, including only the water and oil phases, would have three surfaces across which flow occurs: the water inflow, the oil outflow and the air-oil interface. Treating both the water and oil as incompressible means that if, for **case (a)**, the incoming volume flow rate of water is greater than the outgoing volume flow rate of oil, then the air-oil interface would move up and the air would leave the tank through the top vent.

Conversely, if the incoming volume flow rate of water is less than the outgoing volume flow rate of oil, then the air-oil interface would move lower and the air would enter the tank through the vent. For case (a) where the mass flow rates balance, $(\dot{m}_{IN})_{water} = (\dot{m}_{OUT})_{oil}$, we have

$$\rho_{water} A_{IN} \bar{V}_{IN} = \rho_{oil} SG_{oil} A_{OUT} \bar{V}_{OUT} \quad \text{which simplifies to} \quad Q_{IN} = SG_{oil} Q_{OUT}$$

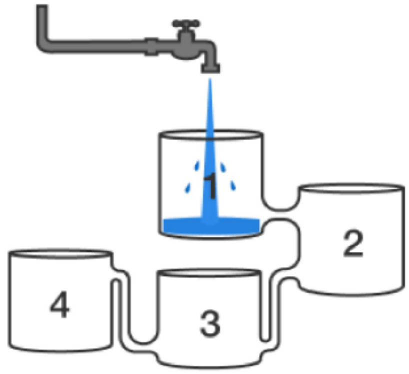
Thus, because $SG_{oil} = 0.9 < 1$, we have $Q_{IN} > Q_{OUT}$ and the air leaves the tank through the top vent. For **case (b)** where $Q_{IN} = Q_{OUT}$ the interface does not move so no air moves either in or out of the top vent.

## Question 16 (4 marks)

Each container in the figure below, has a total volume of one litre. The four containers are positioned so that 60% (by volume) of container one is above the top of container two, 80% (by volume) of container two is above the top of container three and 20% (by volume) of container four is above the top of container three.

If the height of each container is 20 cm, then the center of the hose exiting container one is 2 cm above its base, and 4 cm below the top of container two. The hose exiting container two is 2 cm above its base and it drops to a position that is 4 cm above the base of container three. The hose connecting container three and four exits 4 cm from the base of container three to a position that is 2 cm below the top of container four.

Explain which cylindrical container fills first and the time it takes to fill your container of choice if the initial flow rate out of the tap is 25 cc per second (i.e. 25 $cm^3/s$ ).




Q16 Answer:

Container 3 is the only container that will completely fill. Using the relationship for the volume of a cylinder one can find that the radius of each container is approximately 4 cm. As a consequence, the hose entering container 4 will be above the top of container 3 and, as a consequence, no fluid will flow in the hose connecting container 3 and 4. Thus, the volume that one needs to completely fill container 3 is a combination of the entire volume of container 3 and the 20% (by volume) of container 2 that is below the top of container 3. This volume is 1.2 litres or 1200 cc so the total time required is 1200/25 or 48 seconds.

**Question 17 (4 marks)**

The traffic in a large city is to be studied. Explain how it would be done using:

(a) A Lagrangian approach or viewpoint,
(b) An Eulerian approach or viewpoint.

**Q17 Answer:**

(a) **Lagrangian:** Have observers ride around in cars and record their observations.

(b) **Eulerian:** Position observers on various corners and have them record their observations.

## Section 3: Long Answer Question (Full Solution Required)

***Please provide your answers on separate pages.***

**Question 18 (10 marks)**

A 30° reducing elbow is shown in the Figure below. The fluid is water $(\rho = 999\ [kg/m^3]\ and\ \gamma = 9{,}800\ [N/m^3])$. Assume that there are no energy (i.e. frictional) losses at the elbow.

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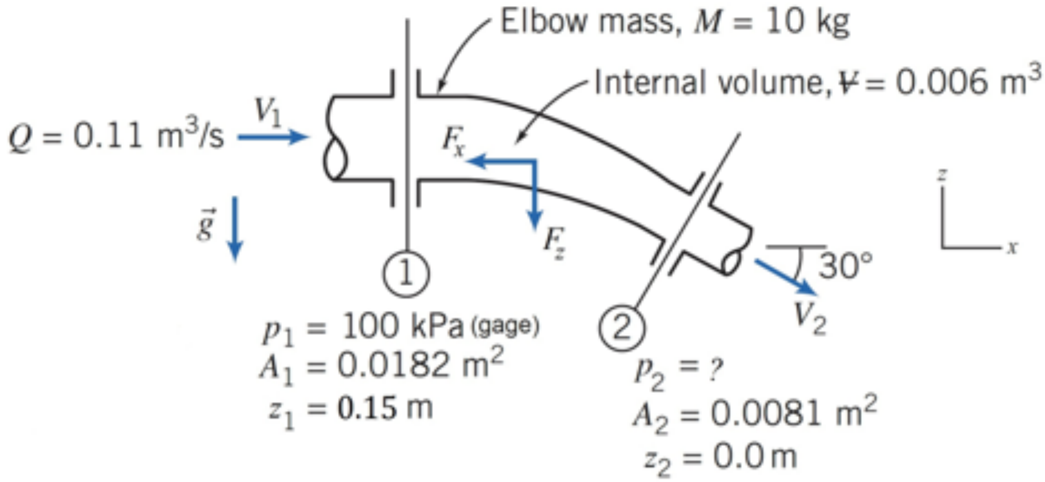


Answer the following questions:

(a) What is average flow speed $V_1$ at pipe cross-section one?
(b) What is average flow speed $V_2$ at pipe cross-section two?
(c) What is the pressure $p_2$ at pipe cross-section two?
(d) To prevent the elbow from moving, what is the horizontal reaction force $F_x$?
(e) To prevent the elbow from moving, what is the vertical reaction force $F_z$?

**Q18 Solutions:**

**Q18 (a):** What is the average flow speed $V_1$?

$$V_1 = \frac{Q}{A_1} = \frac{0.11}{0.0182}$$

$$V_1 = 6.0 \left[\frac{m}{s}\right]$$

**Q18 (b):** What is the average flow speed $V_2$?

$$V_2 = \frac{Q}{A_2} = \frac{0.11}{0.0081}$$

$$V_2 = 13.6 \left[\frac{m}{s}\right]$$

**Q18 (c):** What is the pressure $p_2$ at section two?

$$p_2 = p_1 + \gamma \cdot \left[\frac{{V_1}^2 - {V_2}^2}{2 \cdot g} + (z_1 - z_2)\right]$$

$$p_2 = 100{,}000 + 9800 \cdot \left[\frac{6^2 - 13.6^2}{2 \cdot 9.81} + (0.15 - 0)\right]$$

$$p_2 = 27{,}066\ [Pa]$$

**Q18 (d):** To prevent the elbow from moving, what is the horizontal reaction force $F_x$?

$$-F_x + (p_1 \cdot A_1) - (p_2 \cdot A_2 \cdot \cos\theta) = (\rho \cdot Q \cdot V_2 \cdot \cos\theta) - (\rho \cdot Q \cdot V_1)$$

$$F_x = (p_1 \cdot A_1) - (p_2 \cdot A_2 \cdot \cos\theta) - (\rho \cdot Q) \cdot (V_2 \cdot \cos\theta - V_1)$$

$$F_x = (100{,}000 \cdot 0.0182) - (27{,}066 \cdot 0.0081 \cdot \cos 30°) - (999 \cdot 0.11) \cdot (13.6 \cdot \cos 30° - 6)$$

$$F_x = 995\ [N]$$

**Q18 (e):** To prevent the elbow from moving, what is the vertical reaction force $F_z$?

$$-F_z + (p_2 \cdot A_2 \cdot \sin\theta) - (M \cdot g) - (\gamma \cdot \forall) = -(\rho \cdot Q \cdot V_2 \cdot \sin\theta)$$

$$F_z = (p_2 \cdot A_2 \cdot \sin\theta) - (M \cdot g) - (\gamma \cdot \forall) + (\rho \cdot Q \cdot V_2 \cdot \sin\theta)$$

$$F_z = (27{,}066 \cdot 0.0081 \cdot \sin 30°) - (10 \cdot 9.81) - (9800 \cdot 0.006) + (999 \cdot 0.11 \cdot 13.6 \cdot \sin 30°)$$

$$F_z = 700\ [N]$$