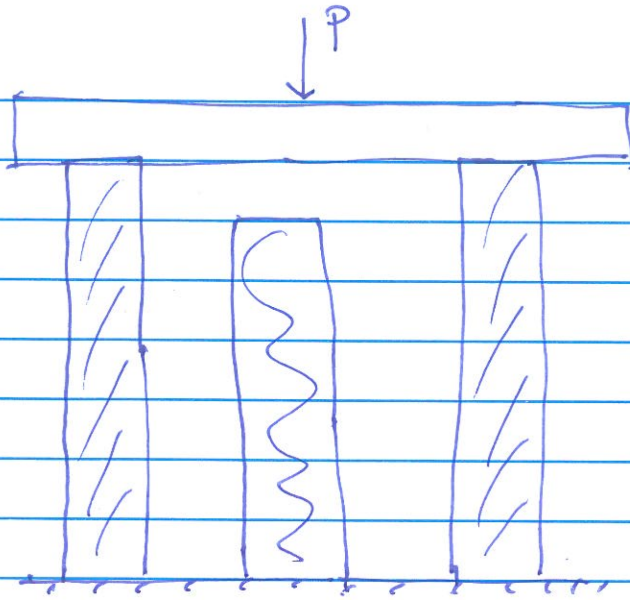
1.

$A_{AL} = 2 \text{ in}^2$
$A_{st} = 4 \text{ in}^2$

$E_{AL} = 10^7 \text{ psi}$
$E_{st} = 30\times10^6 \text{ psi}$

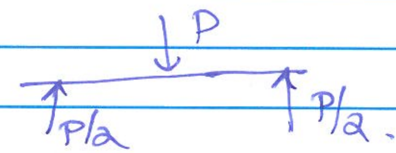
$L_{AL} = 10 \text{ in}$
$L_{st} = 9.995 \text{ in.}$



1a) For the gap to close

$\boxed{3}$

$$\Delta_{AL} = \frac{(P/2)L_{AL}}{A_{AL}E_{AL}} > 0.005 \text{ in}$$ → 2 marks



$$\Rightarrow \Delta_{AL} = \frac{\cancel{100}\times\cancel{10^3}\times \cancel{10}}{2\times 10^{\cancel{7}}\times 2}$$

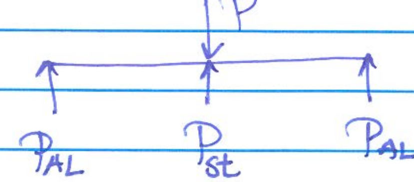
$$\Delta_{AL} = \frac{1}{40} = 0.025 > 0.005 \text{ in.}$$

Hence, the gap will close. → 1 mark

1b) Once the gap closes,

$\boxed{11}$

$$\Sigma F_y = 0 \Rightarrow 2P_{AL} + P_{st} = P \quad \text{-(1)}$$



(Equilibrium Eqns) – 4 marks

the load-displacement eq^{n}s are given as,

$$\left.\begin{aligned}\Delta_{AL} &= \Delta_{St} + 0.005 \\ \frac{P_{AL} L_{AL}}{A_{AL} E_{AL}} &= \frac{P_{St} L_{St}}{A_{St} E_{St}} + 0.005\end{aligned}\right\} \quad -② \quad \rightarrow \underline{4} \text{ marks}$$

Using Eq ① & ②, we get,

$$\frac{(P - P_{St}) L_{AL}}{2A_{AL} \; E_{AL}} = \frac{P_{St} L_{St}}{A_{St} E_{St}} + 0.005 \quad \rightarrow \underline{1} \text{ mark. for calculation}$$

$$\Rightarrow \frac{P L_{AL}}{2A_{AL} E_{AL}} - 0.005 = P_{St}\left[\frac{L_{AL}}{2A_{AL}E_{AL}} + \frac{L_{St}}{A_{St}E_{St}}\right]$$

$$\Rightarrow \frac{100 \times 10^3 \times 10}{2 \times 2 \times 10^7} - 0.005 = P_{St}\left[\frac{10}{2 \times 2 \times 10^7} + \frac{9.995}{4 \times 30 \times 10^6}\right]$$

$$\Rightarrow 0.025 - 0.005 = P_{St}\left[25 \times 10^{-7} + 0.833 \times 10^{-7}\right]$$

$$\Rightarrow P_{St} = \frac{0.02}{3.333 \times 10^{-7}} \Rightarrow \boxed{P_{St} = 60 \text{ kips}} \rightarrow \underline{1} \text{ mark}$$

Hence, using Eq ①,

$$2P_{AL} + 60 = 100$$

$$\Rightarrow \boxed{P_{AL} = 20 \text{ kips}} \rightarrow \underline{1} \text{ mark}$$

1.c) Aluminum rod will shorten by,
[3]

$$\Delta_{AL} = \frac{P_{AL} L_{AL}}{A_{AL} E_{AL}} \rightarrow \underline{\underline{2}} \text{ marks}$$

$$= \frac{\cancel{20} \times \cancel{10^3}^{\,2} \times 10}{\cancel{2} \times \cancel{10^7}^{\,2}}$$

$$\boxed{\Delta_{AL} = 0.01 \text{ in}} \rightarrow \underline{\underline{1}} \text{ mark}$$

1.d) Axial stresses are given as
[3]

$$\sigma_{AL} = E_{AL} \epsilon_{AL} = E_{AL} \frac{\Delta_{AL}}{L_{AL}}$$

$$= \frac{10^7 \times 0.01}{10}$$

$$\boxed{\sigma_{AL} = 10 \text{ ksi}}$$

or

$$\sigma_{AL} = \frac{P_{AL}}{A_{AL}} \rightarrow \underline{\underline{1}} \text{ marks for eq}^n$$

$$= \frac{20 \times 10^3}{2}$$

$$\boxed{\sigma_{AL} = 10 \text{ ksi}} \rightarrow \underline{\underline{1}} \text{ mark}$$

Also,

$$\sigma_{st} = E_{st} \epsilon_{st} = E_{st} \frac{\Delta_{st}}{L_{st}} = E_{st} \frac{(\Delta_{AL} - 0.005)}{L_{st}}$$

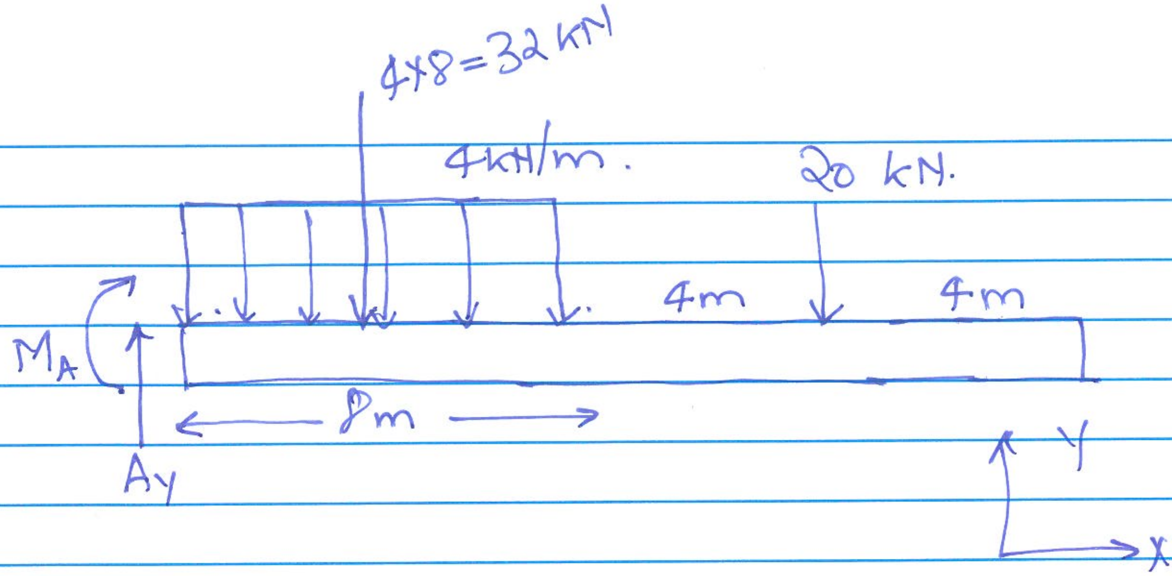
$$= \frac{30 \times 10^6 \times (0.01 - 0.005)}{9.995}$$

$$= \frac{5 \times 30}{9.995} \text{ ksi}$$

$$\boxed{\sigma_{st} = 15 \text{ ksi}} \rightarrow \underline{\underline{1}} \text{ mark}$$

2.

2.a) [1]



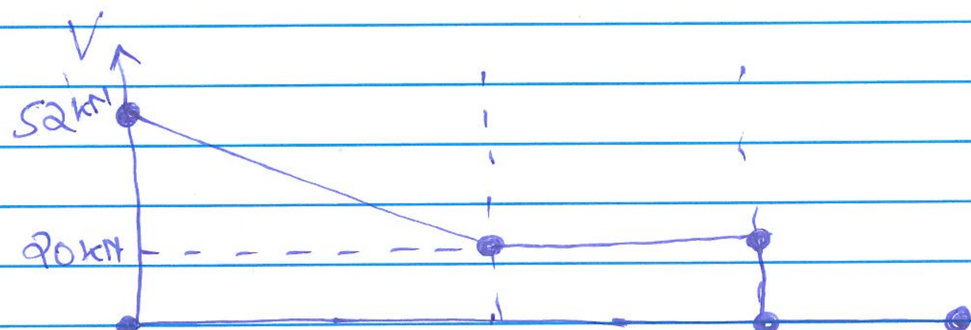
2b) [2] $+\uparrow \sum F_y = 0 \Rightarrow A_y - 32 - 20 = 0$

$\Rightarrow \boxed{A_y = 52 \text{ kN}}$ → 1 mark

$\circlearrowleft + \sum M_A = 0 \Rightarrow -M_A - 32 \times 4 - 20 \times 12 = 0$

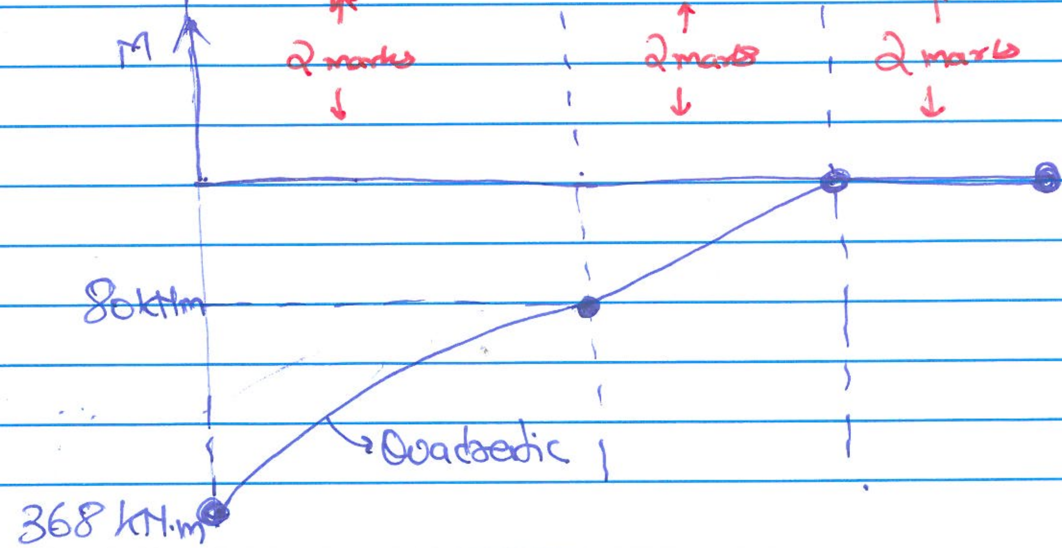
$\Rightarrow \boxed{M_A = -368 \text{ kNm}}$ → 1 mark

2c) [6]

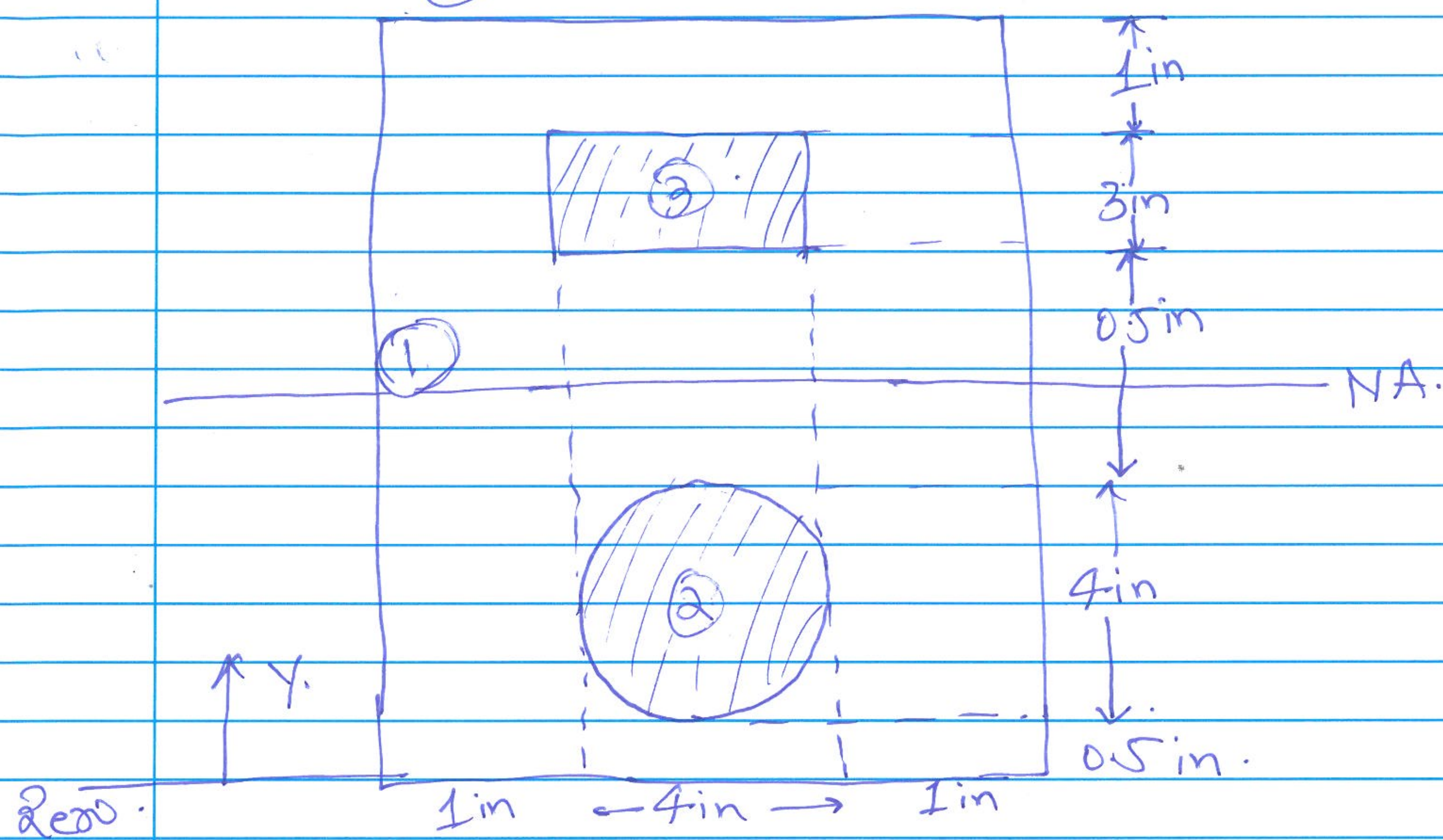


2 marks 2 marks 2 marks

2d) [6]



3. ① = whole rectangle (big one)
② = Circle
③ = rectangle (inside big one)



3.a) Centroid about X-axis is,

6

$$\bar{Y} = \frac{\sum \bar{Y}_i A_i}{\sum A_i} = \frac{A_1\bar{Y}_1 - A_2\bar{Y}_2 - A_3\bar{Y}_3}{A_1 - A_2 - A_3}$$

$$= \frac{(9\times6)(4.5) - (\pi(2)^2)(2.5) - (3\times4)(6.5)}{(9\times6) - \pi 2^2 - (3\times4)}$$

$$= \frac{243 - 31.416 - 78}{54 - 12.566 - 12}$$

$$= \frac{133.584}{29.434} \Rightarrow \boxed{\bar{Y} = 4.538 \text{ in}}$$

$$\Rightarrow \boxed{\bar{Y} = 4.54 \text{ in}}$$

3 b) Moment of inertia, using ∥ axis theorem,

6

$$I = I_1 - I_2 - I_3$$

$$= (\bar{I}_1 + A_1 d_1^2) - (\bar{I}_2 + A_2 d_2^2) - (\bar{I}_3 + A_3 d_3^2)$$

$$I_1 = \bar{I}_1 + A_1 d_1^2 = \frac{1}{12}(6)(9)^3 + (6)(9).(4.54 - 4.5)^2$$

$$= 364.5 + 0.0864$$

$$= 364.586$$

$$I_2 = \bar{I}_2 + A_2 d_2^2 = \frac{\pi\, 2^4}{4} + (\pi(2)^2).(4.54 - 2.5)^2$$

$$= 12.566 + 52.296$$

$$= 64.862 \text{ in}^4$$

$$I_3 = \bar{I}_3 + A_3 d_3^2 = \frac{1}{12}(4)(3)^3 + (4 \times 3)(4.54 - 6.5)^2$$

$$= 9 + 46.099$$

$$= 55.099 \text{ in}^4.$$

$$I = I_1 - I_2 - I_3$$

$$= 364.572 - 64.862 - 55.099$$

$$= 244.611 \text{ in}^4$$

$$\Rightarrow \boxed{I = 244.6 \text{ in}^4}$$

3 c). Maximum bending Stress,

3

$$\sigma_{max} = \frac{MC}{I}$$ → 2 marks

Maximum distance from Neutral axis to the edge of cross-section

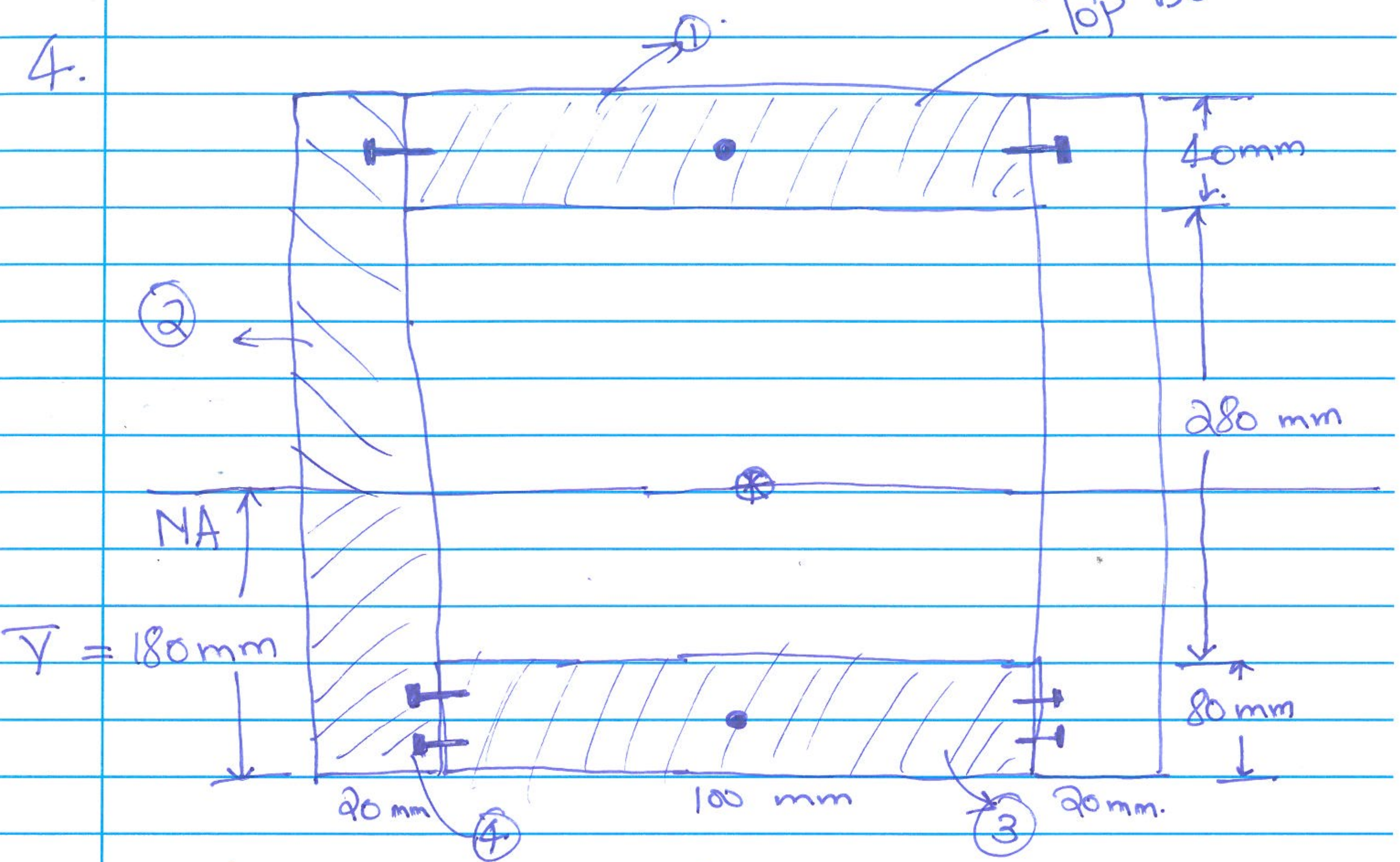
$C = 4.54$ in.

$$\Rightarrow \sigma_{max} = \frac{8 \times 10^3 \times 12 \times 4.54}{244.6} \text{ psi}$$

$$\boxed{\sigma_{max} = 1.78 \text{ KSI}}$$ → 1 mark

$I = 541 \times 10^6\ mm^4$

4.



4 a) For the top board, (Region 1)

1

$$Q = A\bar{Y} = (0.1 \times 0.04)(0.08 + 0.28 + 0.02 - 0.18)$$

$$= 0.004 \times 0.2$$

$$\boxed{Q_{top} = 8 \times 10^{-4}\ m^3 = 8 \times 10^5\ mm^3}$$

4 b) Maximum transverse shear that can be applied without top nails failing is obtained using.

5

$$\tau_{allow} = \frac{V_{max} Q_{top}}{I} = \frac{2F_N}{S}$$ → Because nails are present on both left and right of the board

$$\Rightarrow V_{max} = \frac{F_N I}{Q_{top} S} = \frac{2 \times 500 \times 541 \times 10^6}{8 \times 10^5 \times 50} = 13.525\ kN$$

$$\Rightarrow \boxed{V_{max} = 13.525\ kN}$$

4 c) For the bottom board, (Region 3 on figure)

3

$$Q_{bottom} = A'\bar{y}' = (0.1 \times 0.08)(0.18 - 0.04)$$

$$\boxed{Q_{bottom} = 11.2 \times 10^{-4}\, m^3 = 11.2 \times 10^{5}\, mm^3}$$

4 d) Maximum shear force so as to avoid bottom nails failing is calculated as

5

$$\frac{V_{max}\, Q_{bottom}}{I} = \frac{4 F_N}{S}$$

→ Because there are two sides where nails are and also there is 2 layers of nails on each side.

$$\Rightarrow V_{max} = \frac{4 F_N I}{Q_{bottom}\, S}$$

$$= \frac{4 \times 500 \times 541 \times 10^{6}}{11.2 \times 10^{5} \times 50}$$

$$= 19.321 \text{ kN}$$

$$\Rightarrow \boxed{V_{max} = 19.3 \text{ kN}}$$

4 e) Maximum Q is computed by considering either the section above Neutral axis or below Neutral axis.

$\boxed{3}$

$$Q_{max} = Q_{top} + 2Q_{②}$$

or

$$Q_{max} = Q_{bottom} + 2Q_{④}$$

[see figure for numbering of regions]

Considering all the area below NA,

$$Q_{max} = Q_{bottom} + 2Q_{④}$$

$$= 11.2 \times 10^5 + 2 \times (20 \times 180)\left(\frac{180}{2}\right).$$

$$\boxed{\begin{aligned} Q_{max} &= 17.68 \times 10^5 \, mm^3 \\ &= 17.68 \times 10^{-4} \, m^3 \end{aligned}}$$

4 f) Maximum shear force applied to avoid failure of plywood in shear, ($t = 2 \times 20$ mm) → thickness at NA.

$\boxed{5}$

$$\frac{V_{max} Q_{max}}{It} = \tau_{allow} = 1.5 \times 10^6 \, Pa.$$

$$\Rightarrow V_{max} = \frac{1.5 \times 10^6 \times 541 \times 10^6 \times 10^{-12} \times 0.04}{17.68 \times 10^{-4}}$$

$$= \frac{15 \times 541 \times 4}{1768} \, kN$$

$$= 18.359 \, kN \Rightarrow \boxed{V_{max} = 18.4 \, kN}$$