# SAMPLE TEST QUESTIONS, PART 3 FOR MAT 1348

**Instructions-** These questions have been taken from my old exams from the course MAT 1361, which no longer exists but was similar to MAT 1348. They are an **EXCELLENT** indicator of the sort of questions that will appear on **THE FINAL EXAM!!!!!**. (Note that I used 5 exclamation points. That means I am not kidding.)

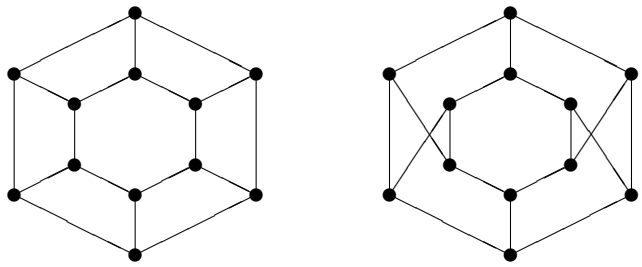
**Question 1**

Let $\mathbf{Z}$ be the set of integers. Define a binary relation $R$ on $\mathbf{Z}$ by $nRm$ if $n^2 - m^2$ is divisible by 3.

1. Prove that $R$ is an equivalence relation.
2. Give 4 elements of the equivalence class of 1.

**Question 2**

Are the following two graphs isomorphic? If so, exhibit an isomorphism, and show that the map is an isomorphism. If they are not isomorphic, explain why not.



**Question 3**

Use induction to prove that for all $n \in \{1, 2, 3, \ldots\}$ that $2^{2n} - 1$ is divisible by 3. Be careful to show each step in your argument clearly.

**Question 4**

- (3 points) A graph with 21 edges has 7 vertices of degree 1, 3 vertices of degree 2, 7 vertices of degree 3 and the rest are of degree 4. How many vertices of degree 4 does it have?
- (7 points) Suppose that $G$ is a graph and that all vertices in $G$ have degree $k$, where $k$ is an odd number. Prove that the total number of edges in $G$ is divisible by $k$.

**Question 5**

1. Let $G = \langle V, E \rangle$ be a graph such that, for all vertices $v$, we have *deg(v)=3*. Suppose also that $|E| = 2|V| - 10$. Find $|V|$ and $|E|$.

2. Now suppose a graph $H$ has 20 edges, what is the least number of vertices it can have?

**Question 6**

Prove, for all $n \in \{0, 1, 2, 3, \ldots\}$ the following by induction:

$$\sum_{i=0}^{n}(4i + 1) = (n + 1)(2n + 1)$$

Be careful to show each step in your argument clearly.

**Question 7**

Let $G =< V, E >$ be an undirected graph with vertex set $V$. Define a binary relation $\mathcal{R}$ on $V$ by:

For all $v, w \in V$, we will say that $v\mathcal{R}w$ if $v = w$ or there is a path from $v$ to $w$.

Prove that $\mathcal{R}$ is an equivalence relation on $V$.

**Question 8**

Suppose a graph $G$ has 17 edges. If each vertex has degree at least 3, i.e. $deg(v) \geq 3$, what is the largest possible number of vertices? Explain.