# MATH 1348 - Lecture 9

Midterm - Monday February 11th

Will Cover K&K 1.1, 1.2, 1.3, Validity of Arguments (Lecture 6), 1.6, 2.1, 2.2

Cell phones
ID's
listeners

## Key Topics

- Propositional Logic
  - what are formulas?
  - translation English → PL
  - truth tables & equivalence
  - tautologies
- Disjunctive Normal Form (Lecture 3)
- Validity of arguments
  - Eng → PL arguments
  - Check Validity

  } See old test questions
- Predicate Logic
  - What are formulas?
  - Logical Equivalence

Note - Validity of predicate formulas will not be on test.

- Validity of Arguments
- How to prove stuff.
  - the 3 methods
  - examples from HW

Ch 2 - Set operations

$\cap, \cup, \overline{(\ )}, \setminus$

identities on p 124

There will be 5 questions

1)
[redacted] DNF
[redacted]
5) Set Identity

<u>One more example of a set identity.</u>

Distributive laws - The book proves

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

I'll prove the other one.

$$A \cup \overset{X}{(B \cap C)} = \overset{Y}{(A \cup B) \cap (A \cup C)}$$

As usual, prove
if $x \in X$, then $x$ ~~B~~ is in $Y$ &
if $y \in Y$, then $y \in X$

Suppose $x \in X$. Since $X$ is a union, then

$x \in A$ or $x \in B \cap C$.

Two cases:

1) If $x \in A$, then $x \in A \cup B$, and $x \in A \cup C$

So $x \in (A \cup B) \cap (A \cup C) = Y$

2) If $x \in B \cap C$, then $x \in B$ and $x \in C$. So

$x \in A \cup B$ and $x \in A \cup C$. So $x \in (A \cup B) \cap (A \cup C) = Y$

In either case, we have shown $x \in Y$.

Now suppose $y \in Y$. So $y \in (A \cup B) \cap (A \cup C)$

So $y \in A \cup B$ and $y \in A \cup C$. If $y \in A$, then

we are done, $y \in A \cup (B \cap C)$. If $y \notin A$, then $y \in B$

and $y \in C$. So $y \in (B \cap C)$ so $y \in X$. $\square$

Moving On - This next material will not be on

Midterm 1.

Section 2.3 Functions studied in calculus,

but what are they?

Def'n 1 p133 Let $A$ and $B$ be sets.

A function $f$ from $A$ to $B$ will be

denoted $f: A \to B$. A function is an assignment of <u>exactly one</u> element of $B$ to every element of $A$. If $b \in B$ is the element assigned to $a \in A$, write $f(a) = b$. $\square$

2 keys

1) every $a \in A$ is assigned a $b \in B$
2) There is only one $b \in B$ assigned to each $a \in A$.

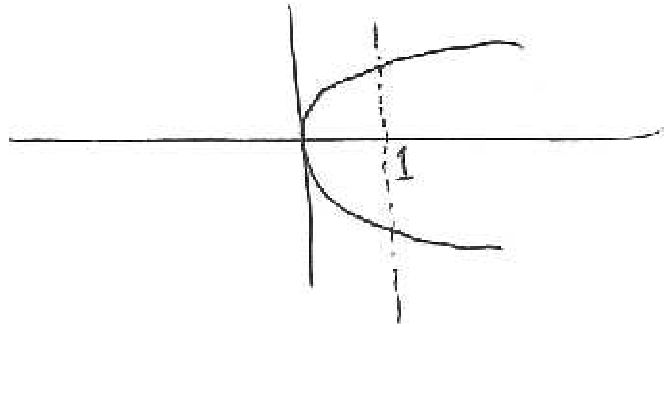
So something can fail to be a function in 2 ways

Ex: $A = \{a, b, c\}$ $B = \{d, e, f\}$

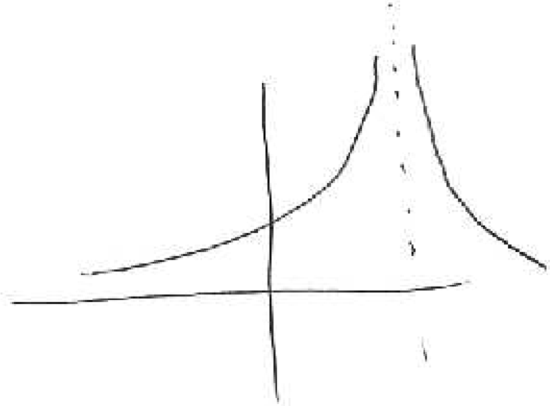
① Suppose $h(a) = d$ and $h(b) = f$.
This fails to be a function since ~~b~~c is not assigned an element

② Suppose $k(a) = d$ $k(b) = e$ $k(c) = d$ and $k(a) = f$
This fails to be a function since $a$ is assigned 2 elements.

Think about graphs $f: \mathbb{R} \to \mathbb{R}$




fails condition 2



fails condition 1

So $f(x) = \frac{1}{(x-1)^2}$ is not a function from $\mathbb{R} \to \mathbb{R}$

It is a function $\mathbb{R} \setminus \{1\} \to \mathbb{R}$.

Key defn's p 136

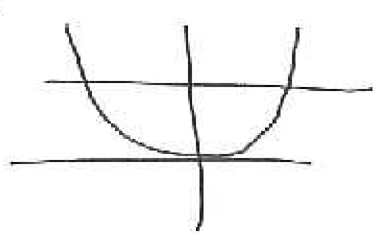
A function $f: A \to B$ is one-to-one (1-1) or injective if

$$f(a) = f(b) \Rightarrow a = b$$

equivalently

$$a \neq b \Rightarrow f(a) \neq f(b)$$

So "different elements of $A$ are sent to different elements of $B$



$f: \mathbb{R} \to \mathbb{R}$
$f(x) = x^2$ is not injective