Lecture 8 - 1348

Next Class

Review for Midterm 1 (Feb 11^{th})

Hints will be given

Now on Website Solutions to Practice Test Questions

Last time Def'ns of Set

Element

Equality of Sets

Subset & Proper Subset

KNOW THESE

Review This

Finite Sets & Cardinality

Bracket Notation

Power Sets (p116)

If X is a set, the power set of X is the set of all subsets of X. Denoted $\mathcal{P}(X)$

EX If $X = \{a,b,c\}$

$$\mathcal{P}(X) = \{ \phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\} \}$$

Thm: If X has n element, $\mathcal{P}(X)$ has 2^n elements. (Proof later)

Cartesian Products

Def'n 8, p117 Ordered n-tuple

$(a_1, a_2, \ldots, a_n)$ is called an ordered n-tuple

Key word being ordered. So note

$$(a_1, a_2) \neq (a_2, a_1) \leftarrow \text{ordered pairs}$$

This is different than sets, where $\{a_1, a_2\} = \{a_2, a_1\}$

Def'n 9, p118 Let A and B be sets. The cartesian product of A and B, denoted $A \times B$, is the set of all ordered pairs (a, b) with $a \in A$, $b \in B$.

So

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$$

Thm: - If $|A| = n$, $|B| = m$, then $|A \times B| = mn$

- If A is any set, then

$$A \times \phi = \phi$$

Def'n 10, p118 The cartesian product of n sets $A_1, A_2, \ldots, A_n$, denoted $A_1 \times A_2 \times \ldots \times A_n$ is the set of all ordered n-tuples $(a_1, a_2, \ldots, a_n)$ with $a_1 \in A_1$, $a_2 \in A_2$ etc

So $A_1 \times A_2 \times \cdots \times A_n =$

$$\{(a_1, a_2, \ldots, a_n) \mid a_1 \in A_1 \text{ and } a_2 \in A_2 \text{ and } \ldots\}$$

Thm on cardinality of this product

$$|A_1 \times A_2 \times \cdots \times A_n| = |A_1| \cdot |A_2| \cdot \cdots |A_n|$$

Section 2.2

Usual defn's of union, intersection, p121

$$A \cup B = \{x \mid x \in A \text{ or } x \in B \text{ (or both)}\}$$
$$A \cap B = \{x \mid x \in A \text{ \underline{and} } x \in B\}$$

Thm: $|A \cup B| = |A| + |B| - |A \cap B|$

Two sets are <u>disjoint</u> if their intersection is empty, i.e. they have no elements in common.

Thm: If A and B are disjoint, then

$$|A \cup B| = |A| + |B|$$

Defn 4, p123

A and B sets. The <u>difference of A and B</u>

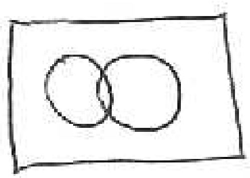
denoted $A-B$ or $A \setminus B$ is the set containing precisely those elements of A which are not in B.

So $A \setminus B = \{x \mid x \in A \wedge x \notin B\}$

Ex: $\underset{\text{rationals}}{\mathbb{R} \setminus \mathbb{Q}}$ = irrationals

$\mathbb{Z} \setminus 2\mathbb{Z}$ = odd #s. (even #s)

<u>Venn Diagrams</u>



shade in $\cup, \cap, \setminus$, see p 122-123

<u>Complement</u> Suppose there is a universal set U "which contains everything"

Then define the complement of A, denoted $\overline{A}$, to be the set of all elements of U, not in A, i.e.

$$\overline{A} = U \setminus A$$
$$= \{x \mid x \in U \wedge x \notin A\}$$

<u>Laws</u> Nice chart on p124

$$\left.\begin{array}{l} A \cup \emptyset = A \\ A \cap U = A \end{array}\right\} \text{Identity}$$

$$\left.\begin{array}{l} A \cup U = U \\ A \cap \emptyset = \emptyset \end{array}\right\} \text{Domination}$$

$$\left.\begin{array}{l} A \cup A = A \\ A \cap A = A \end{array}\right\} \text{Idempotent}$$

$$\overline{\overline{A}} = A \quad \text{Complementation}$$

$$\left.\begin{array}{l} A \cup B = B \cup A \\ A \cap B = B \cap A \end{array}\right\} \text{Comm}$$

$$\left.\begin{array}{l} A \cup (B \cup C) = A \cup (B \cup C) \\ A \cap (B \cap C) = A \cap (B \cap C) \end{array}\right\} \text{Assoc}$$

$$\left.\begin{array}{l} A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \\ A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \end{array}\right\} \text{distributive}$$

$$\left.\begin{array}{l} \overline{A \cup B} = \overline{A} \cap \overline{B} \\ \overline{A \cap B} = \overline{A} \cup \overline{B} \end{array}\right\} \text{de Morgan}$$

These equations should remind you <u>a lot</u> of propositional logic with

$$\cup = \vee$$
$$\cap = \wedge$$
$$\overline{(\ )} = \neg$$

This is <u>not</u> a coincidence

Most are easy to prove, except for

- distributive laws
- de Morgan

Let's prove $\overline{A \cup B} = \overline{A} \cap \overline{B}$

(The book proves $\overline{A \cap B} = \overline{A} \cup \overline{B}$.) on p 125

<u>In general</u>, a good way to prove two sets X and Y are equal is to show

$X \subseteq Y$ if $x \in X$, then $x \in Y$ <u>&</u>

$Y \subseteq X$ if $y \in Y$, then $y \in X$.

Let $x \in \overline{A \cup B}$. So $x \notin A \cup B$. So $x \notin A$ and $x \notin B$.

So $x \in \overline{A}$ and $x \in \overline{B}$. So $x \in \overline{A} \cap \overline{B}$. Thus

$$\overline{A \cup B} \subseteq \overline{A} \cap \overline{B}$$

Let $x \in \overline{A} \cap \overline{B}$. So $x \in \overline{A}$ and $x \in \overline{B}$. So $x \notin A$ and $x \notin B$. Since $x \notin A$ and $x \notin B$, we know $x \notin A \cup B$. So $x \in \overline{A \cup B}$. So $\overline{A} \cap \overline{B} \subseteq \overline{A \cup B}$. ∎

<u>Carefully read</u> Ex 10 & 12. Good test questions

End 2.2

Exam will cover

1.1, 1.2, 1.3, $\underbrace{\text{validity of arguments}}_{\text{part of 1.5}}$, 1.6

2.1, 2.2

Details & HINTS Monday