Lecture 6 - 1348

Last time

Predicate logic, especially English translation

English → Pred Logic

Note - Expect a batch of simple test questions on website over weekend.

Good email question

Translation for "Every comedian is funny"

was $\forall x\ (P(x) \Rightarrow Q(x))$

Does the following also work?

$$\neg \exists x\ (P(x) \wedge \neg Q(x))$$

"It is not the case that there is a comedian who is not funny"

Uses the fact that

$$\forall x\ C(x) \equiv \neg \exists x \neg C(x)$$

This is a special case of de Morgan. Why?

Also last time: Arguments.

In Prop logic (or any logic), an argument looks like:

$$\begin{array}{l} X_1 \\ X_2 \\ \vdots \\ X_n \\ \hline Y \end{array}$$

$X_1, \ldots, X_n$: Premises

The line: "Therefore"

Y: Conclusion

Example from last class

$$\begin{array}{c} (P \wedge T) \Rightarrow A \\ A \Rightarrow D \\ \neg D \wedge T \\ \hline \neg P \end{array}$$

Is this valid? Yes. But need a defn of validity.

Def'n 1 p 64 (first part only)

An argument in prop logic is valid if

For every row of the truth table in which all premises are true, the conclusion is also true

Ex:

$$\begin{array}{c} A \vee \neg B \\ B \\ \hline A \end{array}$$

It ~~is~~ was sunny or I ~~will~~ did not walk to work.
I walked to work. Therefore, it was sunny.

	A	B	B	A ∨ ¬B	A
(1)	T	T	T	T	T
(2)	T	F	F	T	T
(3)	F	T	T	F	F
(4)	F	F	F	T	F

(B, A ∨ ¬B: premises; A: conclusion)

Is the argument valid? YES. The only row that matters is the first row. In rows 2,3,4, at least one of the premises is false.

<u>Ex 2</u>

$$\begin{array}{c} A \wedge (B \Rightarrow C) \\ \neg A \Rightarrow \neg B \\ \neg C \vee A \\ \hline C \Rightarrow B \end{array}$$

Consider:

A	B	C	$A \wedge (B \Rightarrow C)$	$\neg A \Rightarrow \neg B$	$\neg C \vee A$	$C \Rightarrow B$
T	F	T	T	T	T	F

In this row, all the premises are true & conclusion is false. So the argument is not valid. The other rows are irrelevant.

To prove an argument not valid, just find one such row. How did I find this one? Calculate as follows:

I am trying to find a row where all premises are true, and conclusion is false. So $C \Rightarrow B$ must be false. So I must have

$$C = T, B = F$$

Also must have $A = T$ to make first formula true. This is the only possible row. So check it.

This is the <u>shortcut method</u>.

<u>Ex3</u>

$$\begin{array}{c} B \Rightarrow (C \wedge \neg A) \\ C \Rightarrow A \\ \hline A \vee \neg B \end{array}$$

Again, try to find a row where conclusion is false, and premises all true

In this case, I must have

A = F and B = T to make conc false.

Since we <u>must</u> have $C \Rightarrow A$ true, and A is false. We have to have C = F. Why?

We also have to make $B \Rightarrow (C \wedge \neg A)$ true

But I now have A = F, B = T, C = F, and this makes $B \Rightarrow (C \wedge \neg A)$ false. So it is impossible to make both premises true, and conclusion false. So argument is valid.

<u>Note</u> - Each decision was forced, no arbitrary choices.

Ex 4 Is the following valid?

$$\neg A \Rightarrow (C \wedge B)$$
$$C \Rightarrow (B \wedge A)$$

$$A \wedge \neg B$$

Goal - make conc. false and premises true.

Two ways to make $A \wedge \neg B$ false

① Make $A = F$

But this then forces $C = T$ and $B = T$, to make $\neg A \Rightarrow (C \wedge B)$ true.

And this makes $C \Rightarrow (B \wedge A)$ false.

So this guess didn't work.

② Make $B = T$

Then, make $A = C = T$. This makes both premises true

So argument is __not__ valid.

This is all we cover in 1.5.

On to 1.6 "Introduction to proofs"

Have to prove things in this course

So, you have to know what a proof looks like.

Ex 1, p77

If $\underbrace{n \text{ is an odd integer}}_{P}$, then $\underbrace{n^2 \text{ is odd}}_{Q}$.

$$P \Rightarrow Q$$

So, assume P. Assume n is an odd integer.

This means $n = 2m+1$, where m is an integer.

This is key.

After writing out precisely what your assumptions are, the proof is easy.