Lecture 5 - 1348

Last Time - Predicate Logic
Used to deal with

All men are mortal
Every book has a title
Some cats are grey
There is a house in New Orleans

Crucial in logical arguments, cannot be expressed in PL.

Need Predicates, ex:

$T(x)$: x is tall

variable, ranging over a universe

$M(x)$: x is an even #.

One plugs elements of the universe in for variables.
Result will be either true or false.

Ex: $M(3)$ is false
$M(8)$ is true.

If every element satisfies the predicate,
write $\forall x\, M(x)$

This is universal quantification.

Say, "for all x, x satisfies M"

If some element of universe satisfies M, write

$$\exists x\, M(x)$$

Say "There exists an x such that x satisfies M."

This is existential quantification.

Ex: $\mathcal{U} = \mathbb{R}$, the real #'s.

$Q(x): \; x < x+1$

Then $\forall x\, Q(x)$ is true.

$S(x): \; x^2 > 0$

Then $\forall x\, S(x)$ is true.

$L(x): \; x^2 > 3$

$\forall x\, L(x)$ is false, since $L(1)$ is false

But $\exists x\, L(x)$ is true, since $L(2)$ is true.

~~Note~~ Key defn

Defn 3, p39 KNOW THIS

Let X and Y be formulas. X and Y are logically equivalent, denoted $X \equiv Y$, if

In every universe and however the predicates are interpreted, then

X is true if and only if Y is true.

Ex: 1) $\forall x (A(x) \wedge B(x)) \equiv \forall x A(x) \wedge \forall x B(x)$

Last class

2) De Morgan

$$\forall x \neg P(x) \equiv \neg \exists x P(x)$$
$$\neg \forall x P(x) \equiv \exists x \neg P(x).$$

One more "English" version of this.

$\mathcal{U}$ = students in class $C(x)$: x has had a calculus class

It is not the case that everyone has had a calculus class

$\neg \forall x P(x)$

says the same thing as

There is someone who has not had a calculus class

$\exists x \neg P(x)$

HW: Read the proof that these are equiv, p40.

Translating Eng → Pred Logic

All men are mortal

$M(x)$: x is a man

$N(x)$: x is mortal.

For every x, $\forall x\,(M(x) \Rightarrow N(x))$

If x is a man, then x is mortal.

Every comedian is funny

$\forall x\,(C(x) \Rightarrow F(x))$

If x is a comedian, x is funny

Some student in this class has taken calculus

$\exists x\,(S(x) \wedge C(x))$

There is an honest politician

$\exists x\,(H(x) \wedge P(x))$

27 p48

a) A student in your school has lived in Vietnam

$\exists x\,(S(x) \wedge V(x))$

b) There is a student in your school who cannot speak Hindi

$$\exists x (S(x) \land \neg H(x))$$

e) Some student in your school does not play hockey.

$$\exists x (S(x) \land \neg H(x))$$

<u>Arguments</u> Section 1.5 p 63, 64

Here's an <u>argument</u>:

If the phone ~~rings~~ rang and Tim ~~is~~ was home, he ~~will~~ answered it

If Tim answered the phone, he ~~will talk~~ talked to Dianne.

Tim did not talk to Dianne, but Tim was home.

Therefore, the phone didn't ring.

In general, an argument has premises, and a conclusion, separated by the word therefore.

<u>This</u> argument can be translated into PL.

Here:

$$\text{premises} \left\{ \begin{array}{l} (P \land T) \Rightarrow A \\ A \Rightarrow D \\ \neg D \land T \end{array} \right.$$

$$\text{conclusion} \left\{ \begin{array}{l} \hline \neg P \end{array} \right. \quad \leftarrow \text{therefore}$$

Is this argument valid? <u>YES</u>

What is the defn of validity?