Lecture 4 - 1348

Now On Website

PDF of my notes for Lectures 1, 2, 3
Additional Examples of K&K

Course So Far

- Knights & Knaves
- Propositional Logic (PL)
    - translating English into PL
    - logical connectives
    - truth tables
    - disjunctive normal form (DNF)

|| Logical equiv + tautology

Expect several midterm 1 questions from these!

New Topic - Section 1.3 Quantifiers

Aristotle once wrote down the following argument

All lions are fierce
Some lions don't drink coffee
∴ Some fierce creatures do not drink coffee

Socrates is a man.
All men are mortal. } Premises
Therefore Socrates is mortal. } Conclusion

Clearly valid reasoning, but does not translate into PL.

No connective for "All", "Every", "There is" "There are", etc

One requires predicates and <u>predicate logic</u>.

Examples from math:

$E(x)$ = x is even.
(arrow to x:) a variable.

$T(x)$ = x is greater than 3.

$G(x,y)$ = x is greater than y.

These are all predicates. $E(x)$ and $T(x)$ are one-variable. $G(x,y)$ is a 2-variable predicate.

In Pred Logic, variables range over a Universe or domain of discourse.

If I say, "Everybody lives in Canada", this is true if my universe is the people in this room, false if my universe is everybody in the world.

<u>Ex</u>: My universe $\mathcal{U}$ is the set of all real #'s.

$T(x)$ is x is greater than 3.

$T(2)$ is false

$T(4)$ is true

I plugged in an element of universe in for x.

substitution

Suppose the predicate holds for all elements of universe.

Ex: $\mathcal{U} = \mathbb{R}$

$Q(x)$ is $x^2 > 0$.

Then this is true for all x in the universe.

Write $\forall x Q(x)$ Every x satisfies Q

universal quantification.

So $\forall x Q(x)$ is true.

If $\mathcal{U} = \mathbb{C}$, complex #'s.

$\forall x\, Q(x)$ is false.

$T(x)$ is x is greater than 3.

$\forall x\, T(x)$ is false when $\mathcal{U} = \mathbb{R}$.

But ~~[illegible]~~ some elements of $\mathbb{R}$ do satisfy this predicate.

Write $\exists x\, T(x)$

"There is an x ~~such that~~ satisfying T

Predicate logic has

- predicates
- quantifiers
- all connectives from Prop Logic

Sample formulas

$$\forall x\,(P(x) \vee Q(x))$$

$$\exists x\, P(x) \wedge \exists y\, Q(y)$$

$$\neg \exists x\, P(x)$$

etc.

Equivalence

Defn 3, p 39

2 Pred Log formulas are logically equivalent if they have the same truth value, ~~whatever~~ whatever universe is chosen and however the predicates are interpreted

Ex $\forall x\,(P(x) \wedge Q(x)) \equiv \forall x\, P(x) \wedge \forall x\, Q(x)$

Proof: See p 41.

(2) $\neg \forall x P(x) \equiv \exists x \neg P(x)$

p 40

(3) $\forall x \neg P(x) \equiv \neg \exists x P(x)$

Similar

Translations $\mathcal{U}$ = set of all people

All men are mortal

$M(x) =$ x is a man

$N(x) =$ x is mortal

$$\forall x \, (M(x) \Rightarrow N(x))$$

Why?

Every student has studied calculus

$$\forall x (S(x) \Rightarrow C(x))$$

#7 p 47

Every comedian is funny

There is a funny comedian