Lecture 3 - 1348

Last time Truth Tables, be able to calculate the TT of a given formula. Today, Section 1.2

Def'n ~~Two~~ Let X be a formula in PL.

X is a <u>tautology</u> if every it is T in every row of its truth table.

Ex: $p \vee \neg p$

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

$(p \wedge (p \to q)) \to q$

p	q	~~$p \wedge (p \to q)$~~ $p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

<u>Others?</u>

$p \to \neg\neg p$ $p \wedge q \to p$

$\neg\neg p \to p$ $p \wedge q \to q$

$p \leftrightarrow \neg\neg p$ $p \to p \vee q$

$(p \wedge q) \leftrightarrow (q \wedge p)$ $q \to p \vee q$

$(p \vee q) \leftrightarrow (q \vee p)$

Exercise Come up w/ more

Defn
Two formulas X and Y are <u>equivalent</u> or <u>logically equivalent</u> if they have the same truth table.

Ex

$p \wedge q$ is equiv to $q \wedge p$

$p \vee q$ " " " $q \vee p$

$p \wedge (q \wedge r)$ " " " $(p \wedge q) \wedge r$

p " " " $\neg\neg p$

$\neg(p \wedge q)$ " " " $\neg p \vee \neg q$

$\neg(p \vee q)$ " " " $\neg p \wedge \neg q$

$p \wedge (q \vee r)$ " " " $(p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r)$ " " " $(p \vee q) \wedge (p \vee r)$

If you don't believe me, feel free to check!!

$p \to q$ " " " $\neg p \vee q$

$p \to q$ " " " $\neg q \to \neg p$

contrapositive law.

If it is raining, then the game is cancelled

is the same as

If the game is not cancelled, it is not raining.

$p \wedge p$ is the same as p

$p \vee p$ " " " " p

These are called the "idempotent laws".

How do you show two formulas equiv?

Obvious method: Write out the truth tables.

Ex: Show $\neg(p \to q)$ is equiv to $p \wedge \neg q$

p	q	$p \to q$	$\neg(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

p	q	$\neg q$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

SAME

Other method: Use de Morgan and other basic rules:

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$

$$\neg(p \vee q) = \neg p \wedge \neg q$$

$$\neg(p \to q) \equiv \neg(\neg p \vee q) \equiv \neg\neg p \wedge \neg q$$

since $p \to q \equiv \neg p \vee q$

de Morgan

$\equiv p \wedge \neg q$

since $\neg\neg p \equiv p$

Note: This is frequently much easier, especially if there are many variables, e.g.

Show $p \wedge (q \to r) \equiv \neg(\neg p \vee (q \wedge \neg r))$

$\neg(\neg p \vee (q \wedge \neg r)) \overset{dM}{\equiv} \neg\neg p \wedge \neg(q \wedge \neg r)$

$\underset{\neg\neg p \equiv p}{\equiv} p \wedge \neg(q \wedge \neg r) \underset{dM}{\equiv} p \wedge (\neg q \vee \neg\neg r) \underset{\neg\neg r \equiv r}{\equiv} p \wedge (\neg q \vee r)$

$\equiv p \wedge (q \to r)$ $\square$

Note: The only problem with this approach is that it can be sometimes tricky to find the string of equivalences. The TT method is mindless, and ~~always~~ never fails.

Q: 1) How do you show a formula is <u>not</u> a tautology?

2) How do you show two formulas are not equivalent?

Ex: ① $X = p \to (p \wedge (p \to q))$

Is it a tautology? No.

Stupid method. Write out entire truth table. Look for a row with an F.

Smarter method: Just find the row with an F.

In this case, when $p = T$ and $q = F$, X is false

How did I find this?

② $Y = p \wedge ($ really long formula $)$ with 37 variables.

<u>Sol</u>: Any time $p = F$, the formula is false.

③ ~~Are~~ Is $X = p \wedge (p \to (q \wedge r))$ equiv to

$Y = p \to r$

Sol: If $p = T$, $q = F$, $r = T$, then X is false and Y is true.

These are <u>great exam questions</u>!

Q: Suppose I give you the following

p	q	r	X
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

Find a formula X with this as its truth table!

See Q42, p29. Also notes will be on website.

<u>Facts</u> ① One can always, no matter how many variables, find such an X.

② ~~In general, there are lots~~.

In general, there are lots

Method for finding them (It never fails).

This is called the disjunctive normal form

Step 1 - Circle all rows in which X is T.

Step 2. ~~Write~~ For each row, the conjunction of all ~~three~~ (3) atoms, with a negation if the atom is F in that row. With no negation, if it is T in that row. So, here, I get

$p \wedge q \wedge r$

$\neg p \wedge q \wedge r$

$\neg p \wedge \neg q \wedge r$

Step 3 - Take the disjunction of all of these smaller formulas

$$X = (p \wedge q \wedge r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge r)$$

This is it.

Why does it work? Think about it.