Lecture 23 - THE END - 1348

Last Time

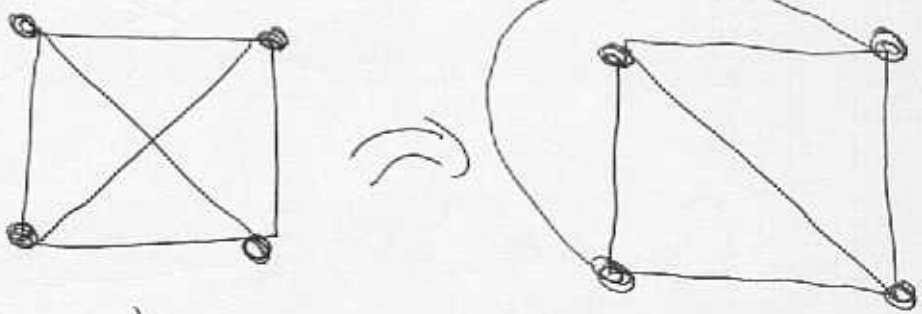
Key Defn: Euler Circuit

Key Thm:

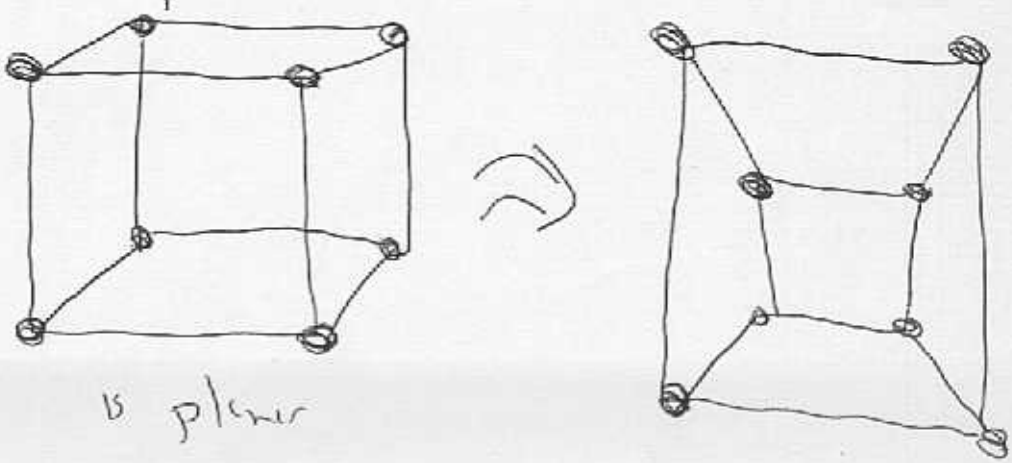
Thm A graph G has an Euler circuit if and only if it is connected, and every vertex has even degree.

Defn: A graph is <u>planar</u> if it can be drawn in such a way that no two edges cross.

Ex 1



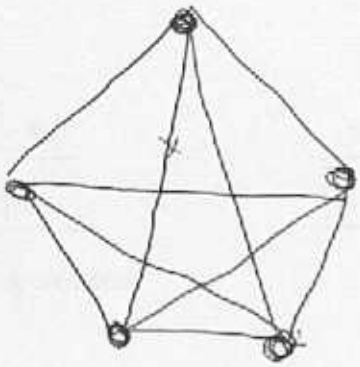
is planar



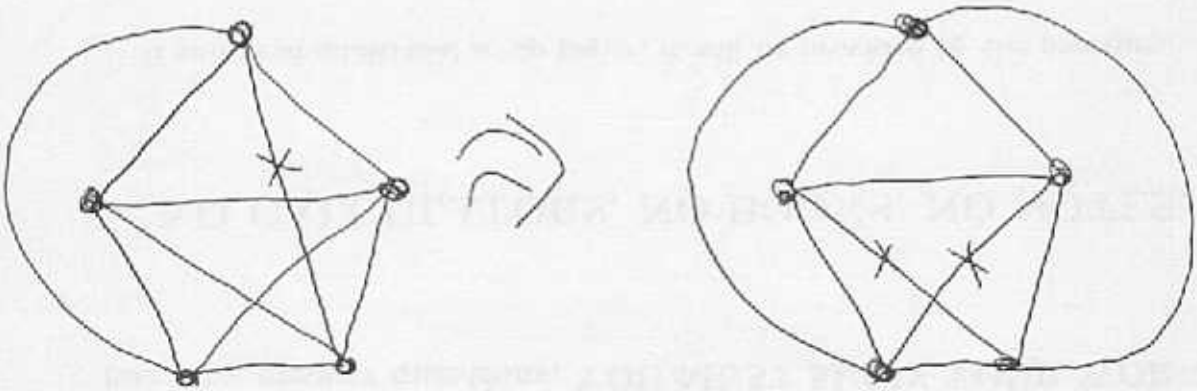
is planar

Is every graph planar? NO. But this is not obvious at all

EX: K_5 is not planar



Let's try to make it planer.



Now I am stuck. Neither of the two x'ed edges can be moved to a planer position.

This isn't really a proof, but is the idea behind it. A real proof requires grad level math, Jordan Curve Thm.

Exercise Make the same argument for $K_{3,3}$

Thm (Kuratowski)

A graph is nonplanar if and only if it "contains" $K_{3,3}$ or K_5

The defn of "contains" is complicated, see p 663

On the test, I could ask:

- Explain why $K_{3,3}$ is not planar
- Give a graph, and say show it is planer.

THE END

Final Exam

9 Questions

2 logic Questions

2 on relations, functions, [redacted] thm

2 on permutations, combinations, binomial [redacted]
in induction

3 on graph theory

Definitely on test

- Induction question
- graph isomorphism question
- predicate logic translation
  English $\Longleftrightarrow$ PL formulas

Topics to study

Logic

- Propositional Logic
  - formulas
  - truth tables
  - validity of an argument
  - tautologies
- Predicate Logic
  - translation

Relations, Functions

- binary relations & their properties
  - refl, symm, transitive
  - all others I would give you defn
  - equivalence relations
  - equivalence classes

- functions
  - What is a function
  - composing functions
  - Injective, surjective functions
- Induction
- Binomial Thm
  - calculate the coefficient of...
- Permutations and combinations
  - know the formulas
- Graph Theory
  - degree of a vertex
  - handshake thm
  - graph isomorphism
  - bipartite graphs
  - paths & connectedness
  - Euler Circuits
  - Planarity

Question 3 (15 Points) The following problem is a multiple choice question. Write your answers in the boxes provided on the next page. We will assume that we have a predicate language with the following:
Constants are given by S=*Saxophone*, and J=*John*
Predicates are given by:

- $M(x)$= x is a musician.
- $I(x)$= x is an instrument.
- $P(x, y)$= x plays y.

For parts 1 through 3, choose the predicate logic formula which corresponds to the given English sentence. For parts 4 and 5, choose the English sentence which corresponds to the given predicate formula.

Part 1- There is a musician who does not play the saxophone.

1. $(\exists x)(M(x) \rightarrow \neg P(x, S))$
2. $(\exists x)(M(x) \wedge \neg P(x, S))$
3. $(\exists x)(\neg M(x) \wedge P(x, S))$

Part 2- Only musicians play the saxophone.

1. $(\forall x)(P(x, S) \rightarrow M(x))$
2. $(\forall x)(M(x) \rightarrow P(x, S))$
3. $(\forall x)(P(x, S) \wedge M(x))$

Part 3- If John is not a musician, then there is no musician who plays the saxophone.

1. $\neg M(J) \rightarrow (\exists x)(M(x) \rightarrow -P(x, S))$
2. $\neg M(J) \rightarrow (\exists x)(M(x) \wedge \neg P(x, S))$
3. $\neg M(J) \rightarrow (\forall x)(M(x) \rightarrow -P(x, S))$

4. $\neg M(J) \Rightarrow (\neg \exists x)(M(x) \wedge P(x, S))$