Lecture 22 - 1348

Vote on Review Session

Options Wednesday, April 16^{th} 1PM *
Thursday April 17^{th} 1PM
Friday April 18^{th} 1PM

Last Time 9.3 Graph Isomorphism Note It will be on final.
This time 9.4. Let G be a graph So $G = (V, E)$
A path in G is a sequence of vertices V

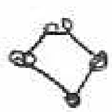
$v_0, v_1, \ldots, v_n$

s.t. each v_i is adjacent to v_{i+1}

We allow paths to have repeated vertices, i.e.

etc.

A path is a circuit if it begins & ends at same vertex



~~Thm:~~ A path is simple if it has no repeated vertices.

A circuit is simple if it has no repeated vertices except starting/ending point.

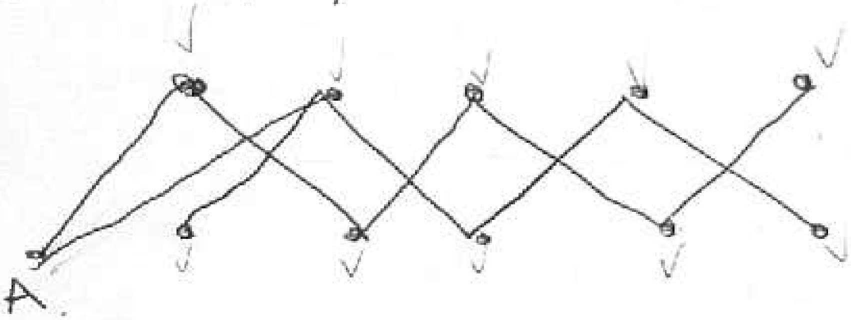
Thm: In a graph G, say that two vertices a and b are related, $a \sim b$, if there is a path from a to b. This is an equivalence relation.

Proof last time. Know it.

Defn: A graph is connected if between any 2 vertices there is a path.

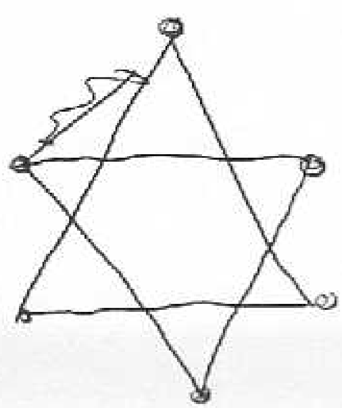
Ex: #4,5 p 630

#4



Note from point A, I can get to every point. Therefore there is a path between any 2 points. Why?

#5



Not connected

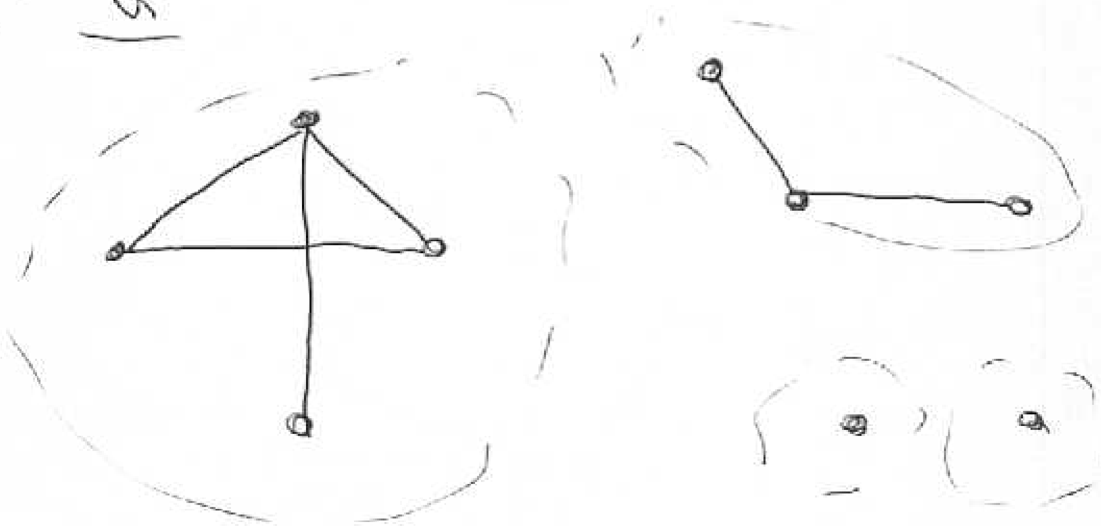
Thm: A graph is connected if and only if for all vertices $a \in V$

$$[a] = \text{equivalence class of } a = V$$

<u>Note</u> Suppose the graph G is not connected. Then if $a \in V$, $[a] \subseteq V$, but is not all of V.

All those vertices which <u>are</u> in $[a]$ form the connected component of a.

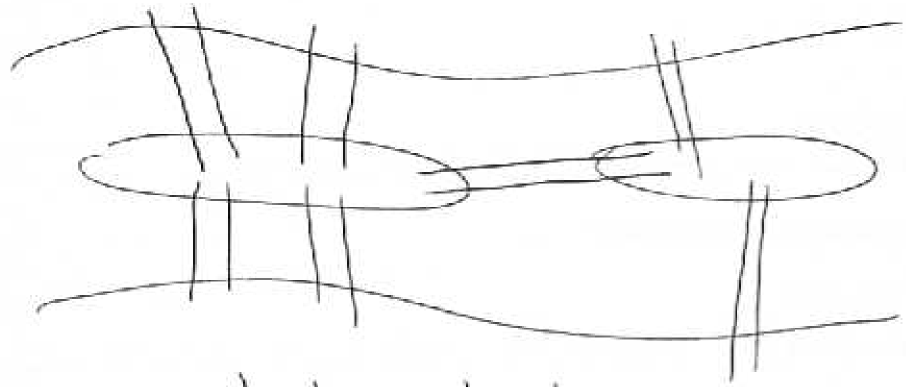
<u>Ex</u>: <u>G</u>



4 connected components.

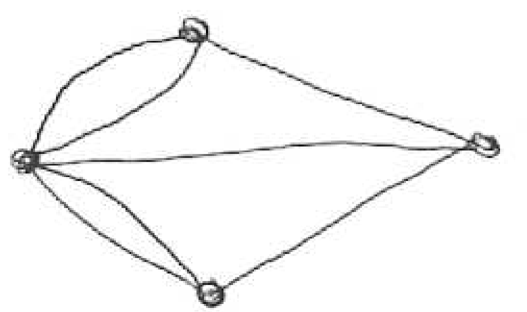
<u>Section 9.5, last section</u>

Konigsberg bridge system



Is there a path through Konigsberg which crosses each bridge exactly once and returns to starting point? You can start wherever you want.

This can be drawn more abstractly as



<u>Note</u> This is a multigraph, not a graph.

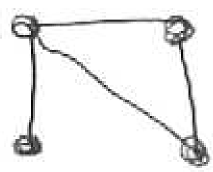
Def'n p 633

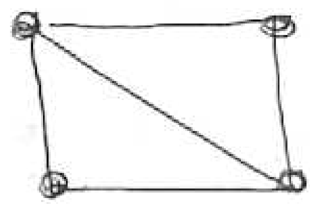
Let G be a connected graph. An <u>Euler circuit</u> is a simple circuit (remember what this means) which contains every ~~vertex~~ edge exactly once.

Note, in particular, an EC starts and ends at same point, since it is a circuit.

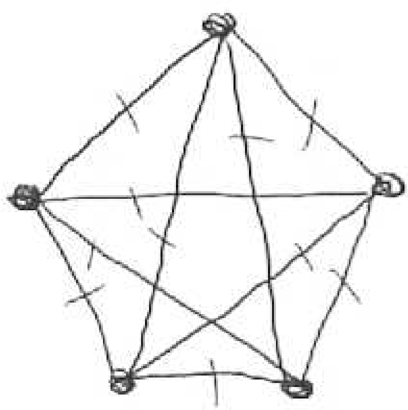
Q: Must a connected graph have a circuit?

No Ex





Both fail. Here's one that works

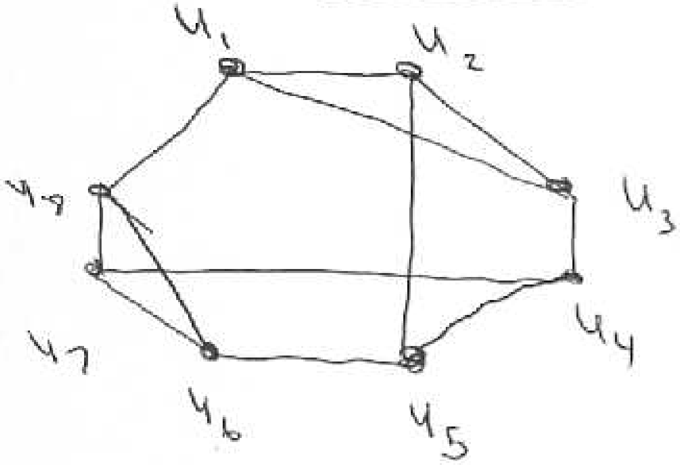


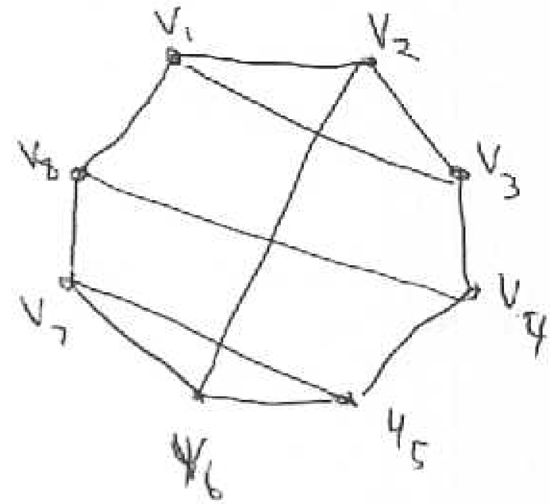
This is K_5

Number edges.

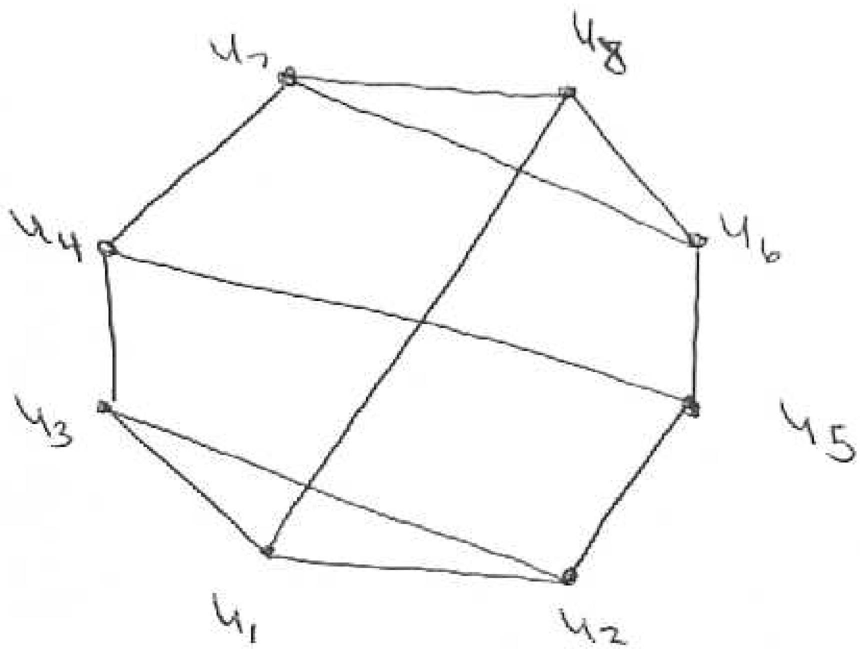
What is the thm?

Graph Iso Example #20, p631





Claim - They are iso.



Add names in this order u_8, u_1 first
Then v_7, u_6 (can go on either side)
Then u_3, u_2 (this is forced)
u_4 is forced
u_5 is forced.