Lecture 19 - 1348

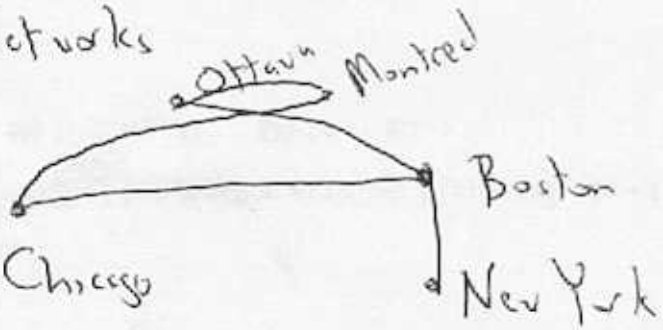
Last Time M. Exams returned at next week's dgd.

Section 4.1 Induction.

Guaranteed it will be on final.

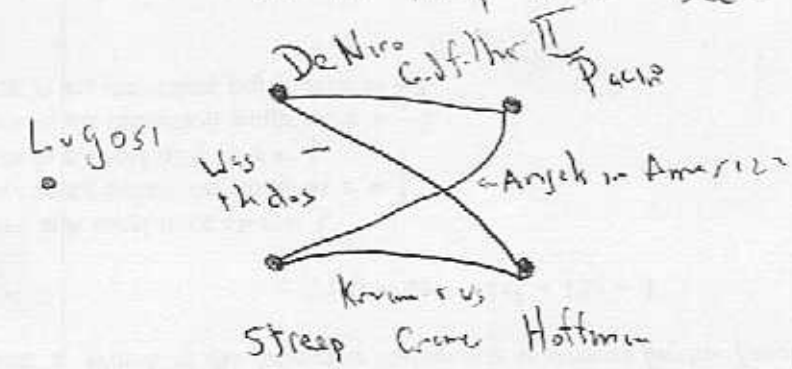
Today Section 9.1 Graph Theory

Ex Flight Networks



An edge between two cities indicates a direct flight.
Cities are represented by vertices.

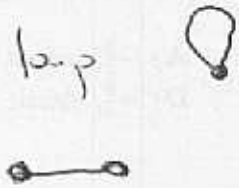
Ex: Vertices could be actors/actresses
Draw an edge if they have been in some movie.



Defn 1 p 589 - A graph $G = (V, E)$ consists of V, a set

of vertices, and E = set of edges. Each edge has either

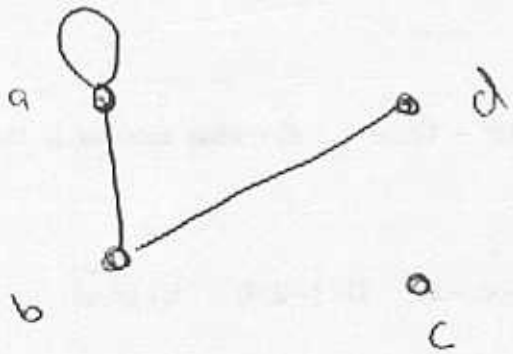
- one end point, in which case it is a loop
- 2 end points



Ex $V = \{a, b, c, d\}$

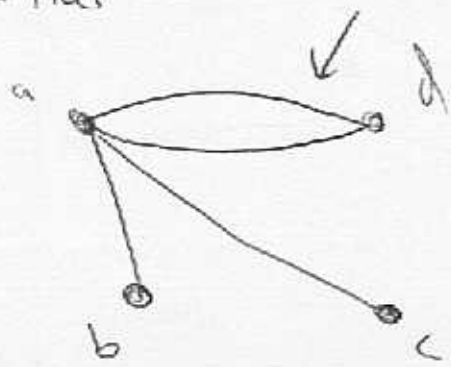
$E = \{\{a,b\}, \{a\}, \{b,d\}\}$

Three edges



Note c has no edges. This is allowed.

A <u>multigraph</u> allows more than one edge between two vertices



2 edges between a and d.

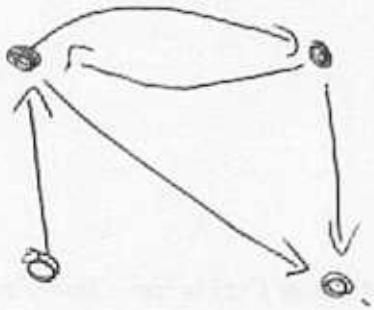
Note - Book not very consistent on this.

For us, always assume

- graph - allow at most one edge between any 2 vertices
- multigraph - allow multiple edges between 2 vertices.

3rd Possibility

Directed graphs, have directed edges



Mostly focus on undirected graphs.

Real life applications?

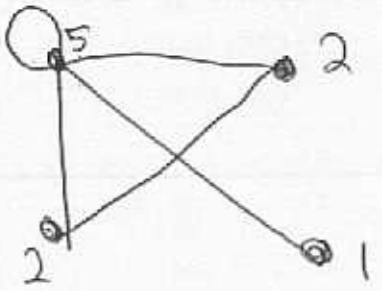
Read Section 9.1 for examples

Section 9.2 Terminology

- Two vertices are adjacent if there is an edge between them
- The degree of a vertex (in an undirected graph) is the # of edges of which

It is an endpoint.

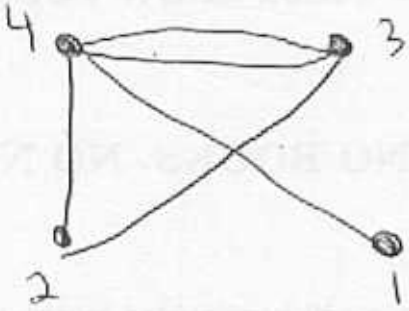
Ex



$e = 5$

$5 + 2 + 2 + 1 = 10$

Note that we count a loop as contributing 2 to the degree.
Works for multigraphs too



$e = 5$

$4 + 3 + 2 + 1 = 10$

Thm 1, p599 (Handshake theorem)
Let G be an undirected graph. Let e be the # of ~~vertices~~ edges. Then

$$2e = \sum_{v \in V} \deg(v)$$

In words, the # of edges is twice the sum of all the degrees. Check our examples

Why does this work?

Each ed is contributing 2 to the total, one for each vertex it has as an endpoint.

Q5, p609

Can a graph exist with 15 vertices, each with degree 5?

No - obvious corollary:

Cor The sum of all the degrees must be even.

Thm 2 The graph G has an even # of odd vertices

Proof is obvious, again, see p 599.

Analogue for directed graphs?

In-degree $\deg^-(v)$

out-degree $\deg^+(v)$

Thm 3 $\sum \deg^-(v) = \sum \deg^+(v) = e$

Complete graphs? How many edges? $\binom{n}{2}$

Q18, p609 Show that in a graph (with no loops) with at least 2 vertices, there must exist 2 vertices with same degree.