Lecture 16 – 1348

Midterm #2 one week from today

Sections covered 2.3, 8.1, 8.5, 5.1, 5.2, 5.3

Topics
- Functions, know defn + injectivity
  surjectivity
- Relations
  know defns & reflexive
  symmetric
  transitive
  Don't need to know others
  Could get a question on
  - checking one of the properties
  - proving something
- Equivalence relations
  - check something is an ER
  - equivalence classes
- Basic Counting questions
  - sum rule, product rule
- PHP and Generalized PHP
- permutations and combinations, know their formulas

Last time, section 5.3

Permutations $n \geq r$

$$P(n,r) = \frac{n!}{(n-r)!}$$

What is it? The # of ways of choosing r elements from an n element set, and arranging them in an order.
This is called an r-permutation

Combinations

$$C(n,r) = \frac{n!}{r!\,(n-r)!}$$

What is it? The # of ways of choosing r elements from an n element set (without regard to order)
The book says "an unordered selection".

Thm: (p359)

$$C(n,r) = C(n,n-r)$$

Pf: $$C(n,n-r) = \frac{n!}{(n-r)!\,(n-(n-r))!}$$

$$= \frac{n!}{(n-r)!\,r!} = C(n,r) \quad \square$$

Non mathematical part?

You will also see notation $\binom{n}{r}$ for $C(n,r)$.

Some problems:

8 p361 In how many different orders can 5 runners finish a race if no ties are allowed? 5!

This is a permutation

Ex 12, p361 How many ways are there to select 5 players from a 10 member tennis club to make a trip to another school?

This is a combination $C(10,5) = \frac{10!}{5!5!}$

Ex 13, p360 How many bit strings of length n contain exactly r 1s?

A: $C(n,r)$. Pick the slots in which to put the 1s. All others get 0s.

13, p361 A group contains n men and n women.

How many ways are there to arrange these people in a row, if men and women alternate?

A: $2(n!)^2$

Where did the 2 come from?

#27, p362 A club has 25 members

A) How many ways are there to choose an executive committee of 4 members? A: $C(25,4) = \frac{25!}{4!21!}$

B) How many ways are there to choose a president, vice-president, secretary, & treasurer? A: $P(25,4)$

#38, p362 How many ways are there to select 12 countries in the United Nations to serve on a council of 12 countries if 3 are selected from a block of 45, 4 are selected from a block of 57 5 are selected from a block of 69?

A: Need post-it rule!

Also order does not matter. So

A: $C(45,3) \cdot C(57,4) \cdot C(69,5)$ □

End New material for midterm #2

Binomial Coefficients

Calculate coefficient of $(x+y)^n$

$$(x+y)^1 = x+y$$
$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$
$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$$

Notice pattern of exponents!
But what about coefficients

$$\begin{array}{ccccccccc} & & & & 1 & & 1 & & \\ & & & 1 & & 2 & & 1 & \\ & & 1 & & 3 & & 3 & & 1 \\ & 1 & & 4 & & 6 & & 4 & & 1 \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \end{array}$$

etc.

This is called <u>Pascal's triangle</u>. Notice the pattern.
Why does it hold?

Thm: (Binomial Thm (Pf next class)

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2$$
$$+ \cdots \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n$$