Lecture 14 - 1348

Now on website

- 2^{nd} Set of Old Exam Questions
  Solutions soon. Some will be done in DGD's
- Next Monday - Review for Second Midterm
  Be there!

Last time - Section 5.1 Counting

Two fundamental principles of counting

Product Rule

If an activity can be done in t successive steps

and step 1 can be done in n_1 ways

" 2 " " " " n_2 ways

⋮

step t " " " n_t ways,

then the # of outcomes of the activity is the product $n_1 n_2 \cdots n_t$.

Sum Rule

Suppose that $X_1, X_2, \ldots X_t$ are sets and

that X_1 has n_1 elements
X_2 has n_2 elements
$\vdots$
X_t has n_t elements.

Suppose that the sets are all disjoint, i.e. $\forall\, i,j$ if $i \neq j$, then $X_i \cap X_j = \emptyset$

Then the # of possible elements that can be selected from X_1 or X_2 or ... or X_t is

$$n_1 + n_2 + \cdots + n_t$$

Ex: An <u>n-bit string</u> is a string of length n made up entirely of 0's and 1's.

<u>Q1:</u> How many 8-bit strings begin with either 101 or 111? (Uses both rule)

$$\boxed{1}\,\boxed{0}\,\boxed{1}\,\boxed{\ }\,\boxed{\ }\,\boxed{\ }\,\boxed{\ }\,\boxed{\ }$$

$$2^5 + 2^5 = 64$$

<u>Q2</u> In how many ways can we pick 2 books from different subjects among 3 math books 5 art books and 2 history books?

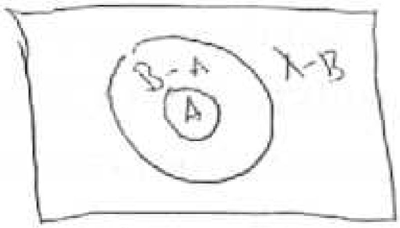
$$5 \cdot 3 + 3 \cdot 2 + 5 \cdot 2 = 31$$

Why?

Q3. Let X be an n-element set. How many ordered pairs (A,B) are there such that A, B are subsets of X and $A \subseteq B$?

Solution: Given an ordered pair (A,B) such that $A \subseteq B \subseteq X$, note that each element in X is in exactly one of the 3 following sets

$$A,\ B-A,\ X-B$$



On the other hand, if we assign each element of X to one of the 3 sets A, $B-A$ or $X-B$, we obtain a unique ordered pair (A,B) of the right type. (We get B by taking the union of A and $B-A$.)

So Answer 3^n. $\square$

5.2 Pigeonhole Principle

Pigeonhole Principle (first version)

If n pigeons fly into k pigeonholes and $k<n$, then some pigeonhole contains

at least 2 pigeons.

Who cares?

Ex1 Suppose that 10 people are in a room. Among these 10 people there are 3 distinct first names, A, B, C and 3 distinct last names D, E, F. Show that there are (at least) 2 people with the same first & last name.

Proof: There are 9 distinct combinations of a first name and a last name. There are 10 different people. Imagine that the people are pigeons. The 9 names are pigeonholes. Giving a person a name corresponds to putting a pigeon in a pigeonhole. □

Second Version of Pigeonhole principle

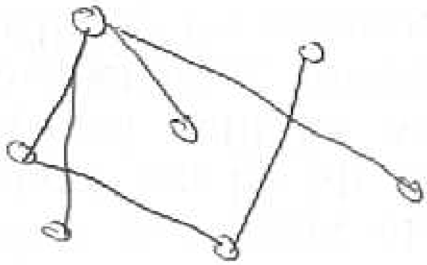
If f is a function from a finite set X to a finite set Y and $|X| > |Y|$, then there are 2 different elements of X, x_1 and x_2 s.t.

$$f(x_1) = f(x_2)$$

$x_1 \neq x_2$

Ex2 If 20 computers are interconnected,

as in



Show that at least 2 computers are connected to the same # of other computers.

Proof: Number the computers $n_1, n_2, \ldots, n_{20}$.

Then note that a computer can be connected to at most 19 other computers. So if we assign to each computer the # of other computers to which it is attached.

So we get a function

$$\{n_1, n_2, \ldots, n_{20}\} \longrightarrow \{0, 1, 2, 3, \ldots, 19\}$$

Both sets have twenty elements, so PHP does not apply.

Is there a trick? Yes. Is it possible to have both 0 and 19 in image? No. Why not?

~~Lecture 14 - 1347~~

Ex4, p348. Show that for every integer n, there is a multiple of n whose decimal expression consists only of 0's and 1's.

Proof: Consider

$1, 11, 111, 1111, 11111$, etc $\qquad \underbrace{11111\ldots1}_{n+1 \text{ 1's}}$

Divide each of these #'s by n.

There are n possible different remainders

$0, 1, 2, \ldots, n-1$.

So two #'s must have same remainder.

Subtract the smaller one from the bigger one, and you are done.

Ex: $n=4$

$1, 11, 111, 1111, 11111$

remainders $1, 3, 3, 3, 3$

$1111 - 111 = 1000$, which is divisible by 4.

Generalized pigeonhole principle

If k is any real #, then $\lceil k \rceil$ is the smallest integer greater than k.

If N objects are placed into k boxes, there is at least one box containing $\lceil N/k \rceil$ objects.