Lecture 12 - 1348

Exams will be returned at DGD's, you must go to correct one. Avg $\frac{17.03}{24} = 71\%$

Last time Section 8.1

Relations A relation from a set A to a set B is a subset $R \subseteq A \times B$

Lots of examples last time.

Notation if $(a,b) \in R$, write aRb.

Properties Reflexive $R \subseteq A \times A$ is reflexive if $\forall a \in A$, aRa (slogan: Everything is related to itself.)

Symmetric $R \subseteq A \times A$ is symmetric if $\forall a, b \in A$, $aRb \Rightarrow bRa$

Transitive

$\forall a, b, c \in A \quad aRb, bRc \Rightarrow aRc$

Consider these relations on $\mathbb{R}$

$a R_0 b$ if $a = b$

$a R_1 b$ if $a \leq b$

$a R_2 b$ if $a > b$

$a R_3 b$ if $a - b > 3$

$a R_4 b$ if $a + b > 3$

$a R_5 b$ if NEVER (empty relation)

Q: Which of these are transitive?

R_0 - if $a = b$ and $b = c$, does $a = c$? Yes

R_1 - if $a \leq b$ and $b \leq c$, ~~does~~ is $a \leq c$? Yes

R_2 if $a > b$ and $b > c$, is $a > c$? Yes

R_3 if $a - b > 3$ and $b - c > 3$, is $a - c > 3$?

Not necessarily $a = 5, b = 4, c = 2$

So R_3 is not transitive

R_4 No $a = 0, b = 4, c = 0$

As usual, to show something is not transitive, or reflexive or symmetric, it suffices to give 1 counter example

There are many properties one can discuss about relations. There are whole books full of them

Sample test question

A relation $R \subseteq A \times A$ is asymmetric if

$aRb \Rightarrow b \not R a$, where $b \not R a$ means

$(b,a) \notin R$.

Note - You do not need to memorize this defn, but you do need to memorize the other 3.

Q: Which of the 6 relations discussed previously is asymmetric?

R_0 - NO, obviously

R_1 - NO, ex $3R_1 3$

R_2 - Yes

R_3 - If $a-b>3$, then $b-a<-3$. So Yes

R_4 No $\quad 2R_4 3$ and $3R_4 2$

R_5 Yes

Section 8.5

Defn 1, p555 A relation on a set A is an equivalence relation if it is reflexive, symmetric & transitive.

Note This is one of the most important defns in the course. Guarantee a test question.

~~Ex~~ Def'n 2. Let A be a set. Let R be an ER on A. If $(a,a') \in R$, write $a \sim a'$.

So the rules for an ER are:

1) $\forall a \in A$, $a \sim a$
2) $\forall a,b \in A$ $a \sim b \Rightarrow b \sim a$
3) $\forall a,b,c \in A$ $a \sim b, b \sim c \Rightarrow a \sim c$

Ex: 1) Let A be any set. Say that aRb if $a=b$. This is the trivial ER

2) Let $\mathbb{R}$ be the real #'s. Define a relation S by sRs' if $s-s'$ is an integer.

Check 3 properties

3) Let $\mathbb{Z}$ be the set of integers. ~~Define~~ Pick n an integer greater than 1.

Say aRb if $a-b$ is divisible by n. This is an ER. This is

<u>integers modulo n.</u>

Defn 3 Let R be an ER on A. Let $a \in A$.
The set of all $a' \in A$ with $a \sim a'$ is called
the equivalence class of a, denoted $[a]_R$ or just $[a]$.

Last example, what are Equivalence classes.

Thm: Let R be an ER on A. The following statements are equivalent.

1) aRa'
2) $[a]=[a']$
3) $[a] \cap [a'] \neq \phi$