Lecture 11 – 1348

Jump to Section 8.1
Before exam, notion of function:

$$f: A \to B$$

A function $f: A \to B$ is an assignment of a unique element $b \in B$ to every $a \in A$. Write $f(a) = b$.

Here is an alternative, equivalent definition:

First recall that $A \times B$ is defined to be

$$A \times B = \{(a,b) \mid a \in A, b \in B\}$$

ordered pair

Defn: A function $f: A \to B$ is a subset, also called f, of $A \times B$

$$f \subseteq A \times B$$

If $(a,b) \in f$, think $f(a) = b$

s.t. 1) $\forall a \in A, \exists b \in B$ with $(a,b) \in f$

2) If $(a,b) \in f$ and $(a,b') \in f$, then $b = b'$.

Make sure you understand why this is equiv to old defn

This leads to more general notion of relation

Defn 1, p519 Let A, B be sets. A **binary relation from A to B** is a subset of $A \times B$.

From this point of view, functions are special cases of relations.

EX: ① See p519-520

Let A be the set of all students in this class
let B be the set of all courses at UO.
Define a relation R from A to B by
$(a,b) \in R$ iff the student a is taking the course b.

Is this a function? NO. Since some students are taking more than one course.

② Let A be the set of all US cities
let B be the set of all US states.

Define R from A to B by
$(a,b) \in R$ if the city a is in the state b.

Is this a function? No, Washington, DC is not in any state

Ex3 Just make one up:

$$A = \{0,1,2\} \quad B = \{a,b\}$$

Define a relation R by

$$(0,a) \in R \quad (1,b) \in R$$

Is this a function? NO, since 2 is not represented

Add $(2,a) \in R$ Is this a function? Yes

Now Add $(0,b) \in R$. Is this a function?

NO, since 0 is represented twice.

Note Relations which are not functions are still interesting, show up in database analysis, as in Ex1.

Ex4 Let $A = \{1,2,3,4\}$

Define a relation from A to A by $(a,a') \in R$ if a divides a'.

Notation - For any relation R from a to b, write

aRb if $(a,b) \in R$.

Ex5+ Relations on $\mathbb{R}$

$a R_1 b$ if $a \leq b$

$a R_2 b$ if $a > b$

$a R_3 b$ if $a = b$

etc

Q: How many relations are there from A to B when A has n elements and B has m elements?

A: $2^{n \times m}$ Why?

Properties of Relations

~~Defn 3, p522~~ A relation R on a set A is <u>reflexive</u> if $aRa \;\forall a \in A$.

Ex: Relations on $\mathbb{R}$

$a R_0 a'$	if	$a = a'$	Yes
$a R_1 a'$	if	$a \le a'$	Yes, it's reflexive
$a R_2 a'$	if	$a < a'$	NO
$a R_3 a'$	if	$a + a' \le 3$	NO
$a R_4 a'$	if	$a - a' \le 3$	YES
$a R_5 a'$		empty relation	NO

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Excellent Test Question!!

Defn 4, p523

A relation R is <u>symmetric</u> if $\forall a, a' \in A$

$$aRa' \Rightarrow a'Ra$$

Go back to our examples. Which are symmetric

0 Yes
1 NO (How to prove?)
2 NO
3 YES
4 NO $4 R 37$, $37 \not R 4$
5 YES!

Defn 4, p 573

R is <u>antysymmetric</u> if

aRb & $bRa \Rightarrow$ ~~aRb~~ $a=b$

Ex

0 YES
1 YES
2 YES
3 NO
4 NO
5 YES