Lecture 10 - 1348

Midterm Monday

5 Questions
Q2 = DNF
Q5 = set identity

full list of topics in lecture 9 on website.

Today 2.3 (Not on test)

Last time Defn of function
1-1 function
onto
} review

$f: X \to Y$ is <u>1-1</u> if $\forall a, b \in X$

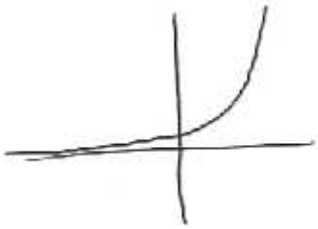
$a \neq b \Rightarrow f(a) \neq f(b)$

$f: X \to Y$ is <u>onto</u> if $\forall y \in Y$, $\exists x \in X$ with

$f(x) = y$.

Think about graphs of functions $f: \mathbb{R} \to \mathbb{R}$

$f(x) = e^x$ is injective, not surjective



$f(x) =$ cubic polynomial

is surjective, not injective

Defn 8, pB38

$f: X \to Y$ is bijective if both 1-1 and onto.

$f: \mathbb{R} \to \mathbb{R}$ defined by ① $f(x) = 2x$ is bijective. Prove it.

Facts about finite sets X, Y finite ② Give finite example

$f: X \to Y$ onto $\Rightarrow |X| \geq |Y|$ ③ $id: X \to X$

$f: X \to Y$ 1-1 $\Rightarrow |X| \leq |Y|$

$f: X \to Y$ bij $\Rightarrow |X| = |Y|$

terrible notation

Defn 9, pB9

$f: A \to B$ bijective. The inverse of f, denoted f^{-1}, is the function $f^{-1}: B \to A$ defined by saying

$f^{-1}(b) =$ the unique $a \in A$ s.t. $f(a) = b$.

It is only because f is bijective that this is well defined.

Ex $f(x) = 2x \Rightarrow f^{-1}(x) = \frac{x}{2}$

$f(x) = x^3 \Rightarrow f^{-1}(x) = \sqrt[3]{x}$

finite example

What about $f(x) = x^2$? What about $g(x) = e^x$?

Composition defined is in calculus

The composition is $g \circ f$

Thm: $f: A \to B$ bijective. $\exists\ f^{-1}: B \to A$ exists.

$f^{-1} \circ f : A \to A$ is $id : A \to A$ $\quad f^{-1} \circ f = id_A$

$f \circ f^{-1} : B \to B$ is $id : B \to B$ $\quad f \circ f^{-1} = id_B$

Pf: Think about it.

Work on finite example

Exercises, p146

16 $f: \mathbb{N} \to \mathbb{N} = \{0, 1, 2, \ldots\}$

1) Give f 1-1, not onto.

$f = x + 1$ or $f(x) = 2x$

2) Onto, not 1-1

$$f(x) = \begin{cases} x-1 & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

3) both 1-1 and onto, but not identity

4) Neither

Q19 Which of following is bijection $f: \mathbb{R} \to \mathbb{R}$ (4)

$f(x) = 2x+1$ Yes

$g(x) = x^2+1$ No

$h(x) = x^3$ Yes

$k(x) = \frac{x^2+1}{x^2+2}$

Q29 f, g 1-1 implies ~~f∘g is 1-1~~ f∘g is one-to-one

onto implies onto.

go through proofs