MATH 1348 - Lecture 1

Go over <u>outline</u>

Usual comments.

Discrete Math - Why study it?

Useful in
- computer algorithms
- programming languages
- searches in large data sets
- communications networks
- cryptography
- computer image analysis,
- et cetera

<u>First topic - Logic</u>, the formal analysis of reasoning
- everyday living
- design and analysis of programming languages
- etc

We are concerned with the truth of <u>propositions.</u>

A proposition is a declarative sentence, i.e. asserts a potential fact. It may be either true or false.

<u>Give Examples</u>

Assumption: A proposition must be either true or false, even if we don't know which.

Propositional logic (PL) was created to analyze propositions.

Introduced using Knights & Knaves

PROPOSITIONAL LOGIC

Book ~~calls it statement logic.~~

~~Analysis of the logic of statements~~ A formal system for ~~the analysis of statements the validity of statements~~

Knights and Knaves

There is an island in the Pacific, called the island of knights and knaves. On this island, there are people called knights, who always tell the truth, and knaves, who always lie.

① We have 3 people on the island, A, B and C. Suppose A and B say the following

A – All of us are knaves

B – Exactly one of us is a ~~[illegible]~~ knight.

Can it be determined which A is?
Can it be determined which B is?
Can it be determined which C is?

Can A be a knight? No.

~~So the statement "All of us are knaves" is false.~~

~~This is a "proof by contradiction."~~

So the statement "All of us are knaves" is false,

So, at least one of them is a knight.

Can B be a knave?

This would lead to the following situation.

A - knave
B - knave
C - knight

Is this possible? No. Why not?

So B is a knight.

What is C? A knave.

A - knave
B - knight
C - knave

Two things to note:

- Proof by contradiction

To prove A is a knave, we assumed A was a knight, and we were led to a contradiction. Thus A must be a knave. Note this is not a visious circle.

- Case analysis.

Each of A, B, C is one of two things. Thus there are 8 possible cases. Our analysis eliminated all but one.

Ex2 - Suppose A says "I am a knave but B isn't".

What are A and B?

A - Knave
B - Knave

4 possibilities

Ex3 Suppose A says "~~Eit~~ I am a knave or B is a knight."

The proposition "I am a knave or B is a knight",
is a compound proposition built up from two basic propositions via the connective or.

To analyze the ~~validity~~ truth of a compound proposition, we must know whether the basic propositions are true. This leads to the notion of truth table.

We will frequently denote propositions by Greek letters

$$\varphi, \psi, \varphi_0, \ldots$$

If φ = "I am a knave"

ψ = "B is a knight"

What A says is "φ or ψ"

In propositional logic, or is denoted by $\vee$.

Our proposition is $\varphi \vee \psi$.

The truth of $\varphi \vee \psi$ depends on the truth of φ, ψ.

φ	ψ	$\varphi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

Notes -

(1) In propositional logic, we will assume that every proposition is either true or false.

(2) $\varphi \vee \psi$ is true when both φ, ψ are true. So this is not "exclusive or", which is only true when exactly one of the two propositions is true.

Back to our example —

A said "I am a knave or B is a knight".

Can A be a knave? If A is an knave, then $\Phi \vee \Psi$ is false. By examining the truth table, we see that Φ, Ψ must both be false. But If A is a knave, then Φ is true. Contradiction.

Conclusion — A is a knight.

Thus $\Phi \vee \Psi$ is true. We know that Φ is false. Hence, by examining the truth table, we conclude that Ψ is true. So B is a knight.

Ex. 4 — Suppose A says

"If B is a knight, then I am a knave".