Version 1

**Question 4.** [6 points] Let $A$ be the matrix

$$A = \begin{pmatrix} 5 & -2 \\ 1 & 3 \end{pmatrix}.$$

(a) Show that the eigenvalues of $A$ are $4+i$ and $4-i$.
(b) Find an eigenvector corresponding to the eigenvalue $4+i$.

**Solution:** (a) The eigenvalues of $A$ are the numbers $\lambda$ where the polynomial $det(A-\lambda I)$ equals 0. So let's evaluate that determinant.

$$A - \lambda I = \begin{pmatrix} 5-\lambda & -2 \\ 1 & 3-\lambda \end{pmatrix},$$

so

$$det(A-\lambda I) = (5-\lambda)(3-\lambda) - 1(-2) = \lambda^2 - 8\lambda + 17.$$

The roots of this polynomial are

$$\lambda = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 * 17}}{2} = \frac{8 \pm \sqrt{-4}}{2} = 4 \pm i,$$

as claimed.

(b) The eigenvector corresponding to an eigenvalue $\lambda$ is a nonzero solution $v$ to $(A-\lambda I)v = \mathbf{0}$. Here $\lambda$ is $4+i$, so

$$A - \lambda I = \begin{pmatrix} 1-i & -2 \\ 1 & -1-i \end{pmatrix}.$$

We find the solutions by row-reducing the augmented matrix

$$\left(\begin{array}{cc|c} 1-i & -2 & 0 \\ 1 & -1-i & 0 \end{array}\right)$$

For simplicity we switch the first two rows, R1 $\leftrightarrow$ R2, to get

$$\left(\begin{array}{cc|c} 1 & -1-i & 0 \\ 1-i & -2 & 0 \end{array}\right)$$

We clear the bottom left entry by subtracting $1-i$ times the first row from the second row, R2 $-(1-i)$R1: (notice that $(1-i)*(-1-i) = -1-i+i-1 = -2$)

$$\left(\begin{array}{cc|c} 1 & -1-i & 0 \\ 0 & 0 & 0 \end{array}\right)$$

The second column has no leading entry, so we can set $x_2 = t$, with $t$ arbitrary. The first row gives the equation $x_1 + (-1-i)t = 0$, so $x_1 = (1+i)t$. The general solution is then

$$v = \begin{pmatrix} (1+i)t \\ t \end{pmatrix} = t \begin{pmatrix} 1+i \\ 1 \end{pmatrix}.$$

We'll just pick $t = 1$ to get

$$v = \begin{pmatrix} 1+i \\ 1 \end{pmatrix}$$

as an eigenvector corresponding to the eigenvalue $4+i$. We can confirm that $Av = \lambda v$:

$$Av = \begin{pmatrix} 5 & -2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1+i \\ 1 \end{pmatrix} = \begin{pmatrix} 5(1+i)-2 \\ 1(1+i)+3(1) \end{pmatrix} = \begin{pmatrix} 3+5i \\ 4+i \end{pmatrix},$$
$$\lambda v = (4+i) \begin{pmatrix} 1+i \\ 1 \end{pmatrix} = \begin{pmatrix} (4+i)(1+i) \\ (4+i)(1) \end{pmatrix} = \begin{pmatrix} 3+5i \\ 4i \end{pmatrix}.$$

Version 2

**Question 4.** [6 points] Let $A$ be the matrix

$$A = \begin{pmatrix} 5 & 13 \\ -2 & 3 \end{pmatrix}.$$

(a) Show that the eigenvalues of $A$ are $4+5i$ and $4-5i$.
(b) Find an eigenvector corresponding to the eigenvalue $4+5i$.

**Solution:** (a) The eigenvalues of $A$ are the numbers $\lambda$ where the polynomial $det(A - \lambda I)$ equals 0. So let's evaluate that determinant.

$$A - \lambda I = \begin{pmatrix} 5-\lambda & 13 \\ -2 & 3-\lambda \end{pmatrix},$$

so

$$det(A - \lambda I) = (5-\lambda)(3-\lambda) - 13(-2) = \lambda^2 - 8\lambda + 41.$$

The roots of this polynomial are

$$\lambda = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 * 41}}{2} = \frac{8 \pm \sqrt{-100}}{2} = 4 \pm 5i,$$

as claimed.

(b) The eigenvector corresponding to an eigenvalue $\lambda$ is a nonzero solution $v$ to $(A-\lambda I)v = \mathbf{0}$. Here $\lambda$ is $4+5i$, so

$$A - \lambda I = \begin{pmatrix} 1-5i & 13 \\ -2 & -1-5i \end{pmatrix}.$$

We find the solutions by row-reducing the augmented matrix

$$\left(\begin{array}{cc|c} 1-5i & 13 & 0 \\ -2 & -1-5i & 0 \end{array}\right)$$

For simplicity we switch the first two rows, R1 $\leftrightarrow$ R2, to get

$$\left(\begin{array}{cc|c} -2 & -1-5i & 0 \\ 1-5i & 13 & 0 \end{array}\right)$$

Multiply the first row by $-\frac{1}{2}$ to get:

$$\left(\begin{array}{cc|c} 1 & \frac{1}{2}+\frac{5}{2}i & 0 \\ 1-5i & 13 & 0 \end{array}\right)$$

We clear the bottom left entry by subtracting $1-5i$ times the first row from the second row, R2 $-$ $(1-5i)$R1: (notice that $(1-5i)*(\frac{1}{2}+\frac{5}{2}i) = \frac{1}{2}+\frac{5}{2}i-\frac{5}{2}i+\frac{25}{2} = 13$)

$$\left(\begin{array}{cc|c} 1 & \frac{1}{2}+\frac{5}{2}i & 0 \\ 0 & 0 & 0 \end{array}\right)$$

The second column has no leading entry, so we can set $x_2 = t$, with $t$ arbitrary. The first row gives the equation $x_1 + (\frac{1}{2}+\frac{5}{2}i)t = 0$, so $x_1 = -(\frac{1}{2}+\frac{5}{2}i)t$. The general solution is then

$$v = \begin{pmatrix} -(\frac{1}{2}+\frac{5}{2}i)t \\ t \end{pmatrix} = t\begin{pmatrix} -\frac{1}{2}-\frac{5}{2}i \\ 1 \end{pmatrix}.$$

We'll pick $t = 2$ (though picking $t = 1$ works just as well, this choice gives a simpler eigenvector) to get

$$v = \begin{pmatrix} -1-5i \\ 2 \end{pmatrix}$$

as an eigenvector corresponding to the eigenvalue $4+5i$. We can confirm that $Av = \lambda v$:

$$Av = \begin{pmatrix} 5 & 13 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} -1-5i \\ 2 \end{pmatrix} = \begin{pmatrix} 5(-1-5i)+13(2) \\ -2(-1-5i)+3(2) \end{pmatrix} = \begin{pmatrix} 21-25i \\ 8+10i \end{pmatrix},$$
$$\lambda v = (4+5i)\begin{pmatrix} -1-5i \\ 2 \end{pmatrix} = \begin{pmatrix} (4+5i)(-1-5i) \\ (4+3i)(2) \end{pmatrix} = \begin{pmatrix} 21-25i \\ 8+10i \end{pmatrix}.$$

Version 3

**Question 4.** [6 points] Let $A$ be the matrix

$$A = \begin{pmatrix} 5 & 2 \\ -5 & 3 \end{pmatrix}.$$

(a) Show that the eigenvalues of $A$ are $4+3i$ and $4-3i$.
(b) Find an eigenvector corresponding to the eigenvalue $4+3i$.

**Solution:** (a) The eigenvalues of $A$ are the numbers $\lambda$ where the polynomial $det(A-\lambda I)$ equals 0. So let's evaluate that determinant.

$$A - \lambda I = \begin{pmatrix} 5-\lambda & 2 \\ -5 & 3-\lambda \end{pmatrix},$$

so

$$det(A-\lambda I) = (5-\lambda)(3-\lambda) - 2(-5) = \lambda^2 - 8\lambda + 25.$$

The roots of this polynomial are

$$\lambda = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 * 25}}{2} = \frac{8 \pm \sqrt{-36}}{2} = 4 \pm 3i,$$

as claimed.
(b) The eigenvector corresponding to an eigenvalue $\lambda$ is a nonzero solution $v$ to $(A - \lambda I)v = \mathbf{0}$. Here $\lambda$ is $4 + 3i$, so

$$A - \lambda I = \begin{pmatrix} 1-3i & 2 \\ -5 & -1-3i \end{pmatrix}.$$

We find the solutions by row-reducing the augmented matrix

$$\left(\begin{array}{cc|c} 1-3i & 2 & 0 \\ -5 & -1-3i & 0 \end{array}\right)$$

For simplicity we switch the first two rows, R1 $\leftrightarrow$ R2, to get

$$\left(\begin{array}{cc|c} -5 & -1-3i & 0 \\ 1-3i & 2 & 0 \end{array}\right)$$

Multiply the first row by $-\frac{1}{5}$ to get:

$$\left(\begin{array}{cc|c} 1 & \frac{1}{5}+\frac{3}{5}i & 0 \\ 1-3i & 2 & 0 \end{array}\right)$$

We clear the bottom left entry by subtracting $1 - 3i$ times the first row from the second row, R2 $-$ $(1 - 3i)$R1: (notice that $(1 - 3i) * (\frac{1}{5} + \frac{3}{5}i) = \frac{1}{5} + \frac{3}{5}i - \frac{3}{5}i + \frac{9}{5} = 2$)

$$\left(\begin{array}{cc|c} 1 & \frac{1}{5}+\frac{3}{5}i & 0 \\ 0 & 0 & 0 \end{array}\right)$$

The second column has no leading entry, so we can set $x_2 = t$, with $t$ arbitrary. The first row gives the equation $x_1 + (\frac{1}{5} + \frac{3}{5}i)t = 0$, so $x_1 = -(\frac{1}{5} + \frac{3}{5}i)t$. The general solution is then

$$v = \begin{pmatrix} -(\frac{1}{5}+\frac{3}{5}i)t \\ t \end{pmatrix} = t \begin{pmatrix} -\frac{1}{5}-\frac{3}{5}i \\ 1 \end{pmatrix}.$$

We'll pick $t = -5$ (though picking $t = 1$ works just as well, this choice gives a simpler eigenvector) to get

$$v = \begin{pmatrix} 1+3i \\ -5 \end{pmatrix}$$

as an eigenvector corresponding to the eigenvalue $4 + 3i$. We can confirm that $Av = \lambda v$:

$$Av = \begin{pmatrix} 5 & 2 \\ -5 & 3 \end{pmatrix} \begin{pmatrix} 1+3i \\ -5 \end{pmatrix} = \begin{pmatrix} 5(1+3i)+2(-5) \\ -5(1+3i)+3(-5) \end{pmatrix} = \begin{pmatrix} -5+15i \\ -20-15i \end{pmatrix},$$

$$\lambda v = (4+3i) \begin{pmatrix} 1+3i \\ -5 \end{pmatrix} = \begin{pmatrix} (4+3i)(1+3i) \\ (4+3i)(-5) \end{pmatrix} = \begin{pmatrix} -5+15i \\ -20-15i \end{pmatrix}.$$

Version 4

**Question 4.** [6 points] Let $A$ be the matrix

$$A = \begin{pmatrix} 5 & -10 \\ 1 & 3 \end{pmatrix}.$$

(a) Show that the eigenvalues of $A$ are $4+3i$ and $4-3i$.
(b) Find an eigenvector corresponding to the eigenvalue $4+3i$.

**Solution:** (a) The eigenvalues of $A$ are the numbers $\lambda$ where the polynomial $det(A-\lambda I)$ equals 0. So let's evaluate that determinant.

$$A - \lambda I = \begin{pmatrix} 5-\lambda & -10 \\ 1 & 3-\lambda \end{pmatrix},$$

so

$$det(A-\lambda I) = (5-\lambda)(3-\lambda) - 1(-10) = \lambda^2 - 8\lambda + 25.$$

The roots of this polynomial are

$$\lambda = \frac{-(-8) \pm \sqrt{(-8)^2 - 4*25}}{2} = \frac{8 \pm \sqrt{-36}}{2} = 4 \pm 3i,$$

as claimed.

(b) The eigenvector corresponding to an eigenvalue $\lambda$ is a nonzero solution $v$ to $(A-\lambda I)v = \mathbf{0}$. Here $\lambda$ is $4+3i$, so

$$A - \lambda I = \begin{pmatrix} 1-3i & -10 \\ 1 & -1-3i \end{pmatrix}.$$

We find the solutions by row-reducing the augmented matrix

$$\left(\begin{array}{cc|c} 1-3i & -10 & 0 \\ 1 & -1-3i & 0 \end{array}\right)$$

For simplicity we switch the first two rows, R1 $\leftrightarrow$ R2, to get

$$\left(\begin{array}{cc|c} 1 & -1-3i & 0 \\ 1-3i & -10 & 0 \end{array}\right)$$

We clear the bottom left entry by subtracting $1-3i$ times the first row from the second row, R2 $-(1-3i)$R1: (notice that $(1-3i)*(-1-3i) = -1-3i+3i-9 = -10$)

$$\left(\begin{array}{cc|c} 1 & -1-3i & 0 \\ 0 & 0 & 0 \end{array}\right)$$

The second column has no leading entry, so we can set $x_2 = t$, with $t$ arbitrary. The first row gives the equation $x_1 + (-1-3i)t = 0$, so $x_1 = (1+3i)t$. The general solution is then

$$v = \begin{pmatrix} (1+3i)t \\ t \end{pmatrix} = t\begin{pmatrix} 1+3i \\ 1 \end{pmatrix}.$$

We'll just pick $t = 1$ to get

$$v = \begin{pmatrix} 1+3i \\ 1 \end{pmatrix}$$

as an eigenvector corresponding to the eigenvalue $4+3i$. We can confirm that $Av = \lambda v$:

$$Av = \begin{pmatrix} 5 & -10 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1+3i \\ 1 \end{pmatrix} = \begin{pmatrix} 5(1+3i)-10 \\ 1(1+3i)+3(1) \end{pmatrix} = \begin{pmatrix} -5+15i \\ 4+3i \end{pmatrix},$$
$$\lambda v = (4+3i) \begin{pmatrix} 1+3i \\ 1 \end{pmatrix} = \begin{pmatrix} (4+3i)(1+3i) \\ (4+3i)(1) \end{pmatrix} = \begin{pmatrix} -5+15i \\ 4+3i \end{pmatrix}.$$

Version 5:

**Question 4.** [6 points] Let $A$ be the matrix

$$A = \begin{pmatrix} 5 & -1 \\ 17 & 3 \end{pmatrix}.$$

(a) Show that the eigenvalues of $A$ are $4+4i$ and $4-4i$.
(b) Find an eigenvector corresponding to the eigenvalue $4+4i$.

**Solution:** (a) The eigenvalues of $A$ are the numbers $\lambda$ where the polynomial $det(A - \lambda I)$ equals 0. So let's evaluate that determinant.

$$A - \lambda I = \begin{pmatrix} 5-\lambda & -1 \\ 17 & 3-\lambda \end{pmatrix},$$

so

$$det(A - \lambda I) = (5-\lambda)(3-\lambda) - (-1)17 = \lambda^2 - 8\lambda + 32.$$

The roots of this polynomial are

$$\lambda = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 * 32}}{2} = \frac{8 \pm \sqrt{-64}}{2} = 4 \pm 4i,$$

as claimed.

(b) The eigenvector corresponding to an eigenvalue $\lambda$ is a nonzero solution $v$ to $(A - \lambda I)v = \mathbf{0}$. Here $\lambda$ is $4+4i$, so

$$A - \lambda I = \begin{pmatrix} 1-4i & -1 \\ 17 & -1-4i \end{pmatrix}.$$

We find the solutions by row-reducing the augmented matrix

$$\left( \begin{array}{cc|c} 1-4i & -1 & 0 \\ 17 & -1-4i & 0 \end{array} \right)$$

For simplicity we switch the first two rows, R1 $\leftrightarrow$ R2, to get

$$\left( \begin{array}{cc|c} 17 & -1-4i & 0 \\ 1-4i & -1 & 0 \end{array} \right)$$

Multiply the first row by $\frac{1}{17}$ to get:

$$\left(\begin{array}{cc|c} 1 & -\frac{1}{17} - \frac{4}{17}i & 0 \\ 1-4i & -1 & 0 \end{array}\right)$$

We clear the bottom left entry by subtracting $1-4i$ times the first row from the second row, R2 $-(1-4i)$R1: (notice that $(1-4i) * (-\frac{1}{17}\frac{4}{17}i) = -\frac{1}{17} - \frac{4}{17}i + \frac{4}{17}i - \frac{16}{17} = -1$)

$$\left(\begin{array}{cc|c} 1 & -\frac{1}{17} - \frac{4}{17}i & 0 \\ 0 & 0 & 0 \end{array}\right)$$

The second column has no leading entry, so we can set $x_2 = t$, with $t$ arbitrary. The first row gives the equation $x_1 + (-\frac{1}{17} - \frac{4}{17}i)t = 0$, so $x_1 = (\frac{1}{17} + \frac{4}{17}i)t$. The general solution is then

$$v = \begin{pmatrix} (\frac{1}{17} + \frac{4}{17}i)t \\ t \end{pmatrix} = t \begin{pmatrix} \frac{1}{17} + \frac{4}{17}i \\ 1 \end{pmatrix}.$$

We'll pick $t = 17$ (though picking $t = 1$ works just as well, this choice gives a simpler eigenvector) to get

$$v = \begin{pmatrix} 1+4i \\ 17 \end{pmatrix}$$

as an eigenvector corresponding to the eigenvalue $4+4i$. We can confirm that $Av = \lambda v$:

$$Av = \begin{pmatrix} 5 & -1 \\ 17 & 3 \end{pmatrix} \begin{pmatrix} 1+4i \\ 17 \end{pmatrix} = \begin{pmatrix} 5(1+4i) + (-1)(17) \\ 17(1+4i) + 3(17) \end{pmatrix} = \begin{pmatrix} -12+20i \\ 68+68i \end{pmatrix},$$
$$\lambda v = (4+4i) \begin{pmatrix} 1+4i \\ 17 \end{pmatrix} = \begin{pmatrix} (4+4i)(1+4i) \\ (4+3i)(17) \end{pmatrix} = \begin{pmatrix} -12+20i \\ 68+68i \end{pmatrix}.$$

version 1

**Question 5.** [3 points] Find the complete solution to the system of equations

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 3 \\ x_1 + 2x_2 - 3x_3 &= -2 \\ 3x_1 + 5x_2 + x_3 &= 1. \end{aligned}$$

Be sure to show all your work.

**Solution:** We construct the augmented matrix from this system of equations, and then row reduce it. The augmented matrix is

$$\left(\begin{array}{ccc|c} 2 & 3 & 4 & 3 \\ 1 & 2 & -3 & -2 \\ 3 & 5 & 1 & 1 \end{array}\right)$$

At this point, the entry in the first column of the second row (1) is simpler than the first-column entry of the first row, so it makes sense to switch rows. We will switch the first two rows, R1 $\leftrightarrow$ R2:

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 2 & 3 & 4 & 3 \\ 3 & 5 & 1 & 1 \end{array}\right),$$

then clear the rest of the first column, with R2 − 2R1 and R3 − 3R1 (replacing the second and third rows, respectively, with these):

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & -1 & 10 & 7 \\ 0 & -1 & 10 & 7 \end{array}\right).$$

Next we replace R3 with R3 − R2, and then we multiply R2 by $-1$, to get

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & 1 & -10 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right),$$

a matrix in row-echelon form. Lastly (this step is not necessary, we can solve from the row-echelon form), we replace R1 with R1 − 2R2 to get

$$\left(\begin{array}{ccc|c} 1 & 0 & 17 & 12 \\ 0 & 1 & -10 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right),$$

There is no leading entry in the third column, so we can set $x_3 = t$ with $t$ arbitrary. From the equations corresponding to the first two rows, we find that the general solution is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 12 - 17t \\ -7 + 10t \\ t \end{pmatrix}$$

version 2

**Question 5.** [3 points] Find the complete solution to the system of equations

$$\begin{aligned} 5x_1 + 8x_2 + 5x_3 &= 4 \\ x_1 + 2x_2 - 3x_3 &= -2 \\ 3x_1 + 5x_2 + x_3 &= 1. \end{aligned}$$

Be sure to show all your work.

**Solution:** We construct the augmented matrix from this system of equations, and then row reduce it. The augmented matrix is

$$\left(\begin{array}{ccc|c} 5 & 8 & 5 & 4 \\ 1 & 2 & -3 & -2 \\ 3 & 5 & 1 & 1 \end{array}\right)$$

At this point, the entry in the first column of the second row (1) is simpler than the first-column entry of the first row, so it makes sense to switch rows. We will switch the first two rows, R1 $\leftrightarrow$ R2:

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 5 & 8 & 5 & 4 \\ 3 & 5 & 1 & 1 \end{array}\right),$$

then clear the rest of the first column, with R2 – 5R1 and R3 – 3R1 (replacing the second and third rows, respectively, with these):

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & -2 & 20 & 14 \\ 0 & -1 & 10 & 7 \end{array}\right).$$

Next we'll switch the second and third rows R2 $\leftrightarrow$ R3:

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & -1 & 10 & 7 \\ 0 & -2 & 20 & 14 \end{array}\right).$$

Next we replace R3 with R3 – 2R2, and then we multiply R2 by $-1$, to get

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & 1 & -10 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right),$$

a matrix in row-echelon form. Lastly (this step is not necessary, we can solve from the row-echelon form), we replace R1 with R1 – 2R2 to get

$$\left(\begin{array}{ccc|c} 1 & 0 & 17 & 12 \\ 0 & 1 & -10 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right),$$

There is no leading entry in the third column, so we can set $x_3 = t$ with $t$ arbitrary. From the equations corresponding to the first two rows, we find that the general solution is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 12 - 17t \\ -7 + 10t \\ t \end{pmatrix}$$

version 3

**Question 5.** [3 points] Find the complete solution to the system of equations

$$\begin{aligned} 4x_1 + 7x_2 - 2x_3 &= -1 \\ x_1 + 2x_2 - 3x_3 &= -2 \\ 3x_1 + 5x_2 + x_3 &= 1. \end{aligned}$$

Be sure to show all your work.

**Solution:** We construct the augmented matrix from this system of equations, and then row reduce it. The augmented matrix is

$$\left(\begin{array}{ccc|c} 4 & 7 & -2 & -1 \\ 1 & 2 & -3 & -2 \\ 3 & 5 & 1 & 1 \end{array}\right)$$

At this point, the entry in the first column of the second row (1) is simpler than the first-column entry of the first row, so it makes sense to switch rows. We will switch the first two rows, R1 $\leftrightarrow$ R2:

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 4 & 7 & -2 & -1 \\ 3 & 5 & 1 & 1 \end{array}\right),$$

then clear the rest of the first column, with R2 − 4R1 and R3 − 3R1 (replacing the second and third rows, respectively, with these):

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & -1 & 10 & 7 \\ 0 & -1 & 10 & 7 \end{array}\right).$$

Next we replace R3 with R3 − R2, and then we multiply R2 by −1, to get

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & 1 & -10 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right),$$

a matrix in row-echelon form. Lastly (this step is not necessary, we can solve from the row-echelon form), we replace R1 with R1 − 2R2 to get

$$\left(\begin{array}{ccc|c} 1 & 0 & 17 & 12 \\ 0 & 1 & -10 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right),$$

There is no leading entry in the third column, so we can set $x_3 = t$ with $t$ arbitrary. From the equations corresponding to the first two rows, we find that the general solution is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 12 - 17t \\ -7 + 10t \\ t \end{pmatrix}$$

version 4

**Question 5.** [3 points] Find the complete solution to the system of equations

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 3 \\ x_1 + 2x_2 - 3x_3 &= -2 \\ -x_1 - x_2 - 7x_3 &= -5. \end{aligned}$$

Be sure to show all your work.

**Solution:** We construct the augmented matrix from this system of equations, and then row reduce it. The augmented matrix is

$$\left(\begin{array}{rrr|r} 2 & 3 & 4 & 3 \\ 1 & 2 & -3 & -2 \\ -1 & -1 & -7 & -5 \end{array}\right)$$

At this point, the entries in the first column of the second and third rows (1 and $-1$, respectively), are simpler than the first-column entry of the first row, so it makes sense to switch rows. We will switch the first two rows, R1 $\leftrightarrow$ R2:

$$\left(\begin{array}{rrr|r} 1 & 2 & -3 & -2 \\ 2 & 3 & 4 & 3 \\ -1 & -1 & -7 & -5 \end{array}\right),$$

then clear the rest of the first column, with $\mathrm{R2} - 2\mathrm{R1}$ and $\mathrm{R3} + \mathrm{R1}$ (replacing the second and third rows, respectively, with these):

$$\left(\begin{array}{rrr|r} 1 & 2 & -3 & -2 \\ 0 & -1 & 10 & 7 \\ 0 & 1 & -10 & -7 \end{array}\right).$$

Next we replace R3 with $\mathrm{R3} + \mathrm{R2}$, and then we multiply R2 by $-1$, to get

$$\left(\begin{array}{rrr|r} 1 & 2 & -3 & -2 \\ 0 & 1 & -10 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right),$$

a matrix in row-echelon form. Lastly (this step is not necessary, we can solve from the row-echelon form), we replace R1 with $\mathrm{R1} - 2\mathrm{R2}$ to get

$$\left(\begin{array}{rrr|r} 1 & 0 & 17 & 12 \\ 0 & 1 & -10 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right),$$

There is no leading entry in the third column, so we can set $x_3 = t$ with $t$ arbitrary. From the equations corresponding to the first two rows, we find that the general solution is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 12 - 17t \\ -7 + 10t \\ t \end{pmatrix}$$

version 5:

**Question 5.** [3 points] Find the complete solution to the system of equations

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 3 \\ x_1 + 2x_2 - 3x_3 &= -2 \\ 5x_1 + 9x_2 - 5x_3 &= -3. \end{aligned}$$

Be sure to show all your work.

**Solution:** We construct the augmented matrix from this system of equations, and then row reduce it. The augmented matrix is

$$\left(\begin{array}{ccc|c} 2 & 3 & 4 & 3 \\ 1 & 2 & -3 & -2 \\ 5 & 9 & -5 & -3 \end{array}\right)$$

At this point, the entry in the first column of the second row (1) is simpler than the first-column entry of the first row, so it makes sense to switch rows. We will switch the first two rows, R1 $\leftrightarrow$ R2:

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 2 & 3 & 4 & 3 \\ 5 & 9 & -5 & -3 \end{array}\right),$$

then clear the rest of the first column, with R2 $-$ 2R1 and R3 $-$ 5R1 (replacing the second and third rows, respectively, with these):

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & -1 & 10 & 7 \\ 0 & -1 & 10 & 7 \end{array}\right).$$

Next we replace R3 with R3 $-$ R2, and then we multiply R2 by $-1$, to get

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & 1 & -10 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right),$$

a matrix in row-echelon form. Lastly (this step is not necessary, we can solve from the row-echelon form), we replace R1 with R1 $-$ 2R2 to get

$$\left(\begin{array}{ccc|c} 1 & 0 & 17 & 12 \\ 0 & 1 & -10 & -7 \\ 0 & 0 & 0 & 0 \end{array}\right),$$

There is no leading entry in the third column, so we can set $x_3 = t$ with $t$ arbitrary. From the equations corresponding to the first two rows, we find that the general solution is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 12 - 17t \\ -7 + 10t \\ t \end{pmatrix}$$