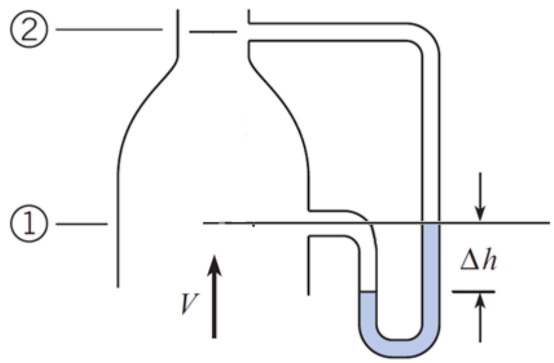
CVG2116 – Introduction to Fluid Mechanics

Assignment 4 **Due Date: 16/02/2018**

1. Air with density of 1.032 $\frac{kg}{m^3}$ is flowing upward in the vertical duct. The area ratio between sections 1 and 2 is $\frac{A_2}{A_1} = 0.5$. Two pressure taps at sections 1 and 2 are connected to a manometer, and manometer measures $\Delta h = 0.049\ m$. Find the velocity at the main duct. Manometer fluid $\gamma_m = 18850.5\ \frac{N}{m^3}$.



Bernoulli's Equation between section 1 and 2:

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

Datum: at section1.

assume section 1 and 2 distance is D: $z_1 = 0, z_2 = D$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} - D = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

$$A_1V_1 = A_2V_2$$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} - D = \frac{v_1^2}{2g}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

Manometer:

$$P_1 + \gamma\,\Delta h - \gamma_m \Delta h - \gamma\,D = P_2$$

$$\frac{P_1}{\gamma} - \frac{P_2}{\gamma} - D = \Delta h \left(\frac{\gamma_m}{\gamma} - 1\right)$$

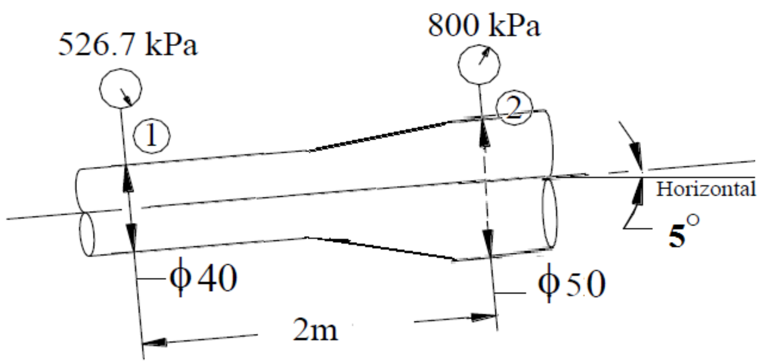
Manometer and Bernoulli's:

$$\Delta h \left(\frac{\gamma_m}{\gamma} - 1\right) = \frac{v_1^2}{2g}\left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

$$(0.049)\left(\frac{18850.5}{1.032 * 9.81} - 1\right) = \frac{v_1^2}{2 * 9.81}\left[(2)^2 - 1\right]$$

$$v_1 = 24.42\frac{m}{s}$$

2. Water (T = 20°C) flows in a pipe inclined at 5° to the horizontal. There is an expansion in the pipe (diameter changes from 40mm to 50mm), and pressure gauges read pressures at section 1 and 2 as shown in the figure below. Determine the flow velocity at section 1 and 2.



Bernoulli's Equation between section 1 and 2:

$$\frac{P_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{P_2}{\gamma} + z_2 + \frac{v_2^2}{2g}$$

$$A_1V_1 = A_2V_2$$

$$V_1\left(\frac{D_1}{D_2}\right)^2 = V_2$$

Datum at z1: $z_1 = 0$

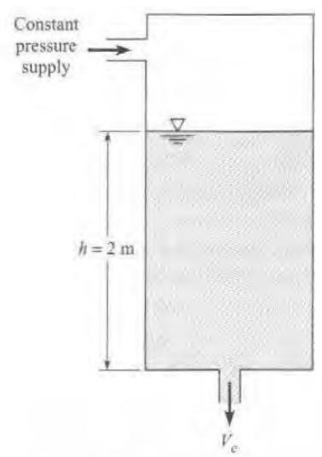
$$z_2 = 2 * \sin\theta$$

$$\frac{v_1^2}{2g}\left(1-\left(\frac{D_1}{D_2}\right)^2\right) = -\frac{P_1}{\gamma} + \frac{P_2}{\gamma} + 2 * \sin\theta$$

$$v_1 = \sqrt{\frac{2g}{\left(1-\left(\frac{D_1}{D_2}\right)^4\right)}\left[-\frac{P_1}{\gamma} + \frac{P_2}{\gamma} + 2 * \sin\theta\right]}$$

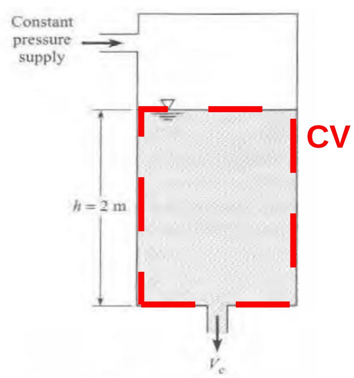
$$v_1 = \sqrt{\frac{2 * 9.81}{\left(1-\left(\frac{40}{50}\right)^4\right)}\left[-\frac{526700}{9810} + \frac{800000}{9810} + 2 * \sin 5\right]} = 30.5m/s$$

3. Water is draining from a pressurized tank as shown in the figure. The exit velocity is given by $v_c = \sqrt{\frac{2P}{\rho} + 2gh}$ where P is the pressure in the tank, ρ is the water density, and h is the water elevation above the outlet. The depth of the water in the tank is $h = 2\ m$ and it has a cross-sectional area of $1\ m^2$ with the exit area of $10\ cm^2$. The pressure in the tank is maintained at $P = 10\ kPa$.



a. Find the time required to empty the tank.
b. Compare the time in part a with the time required if the tank is not pressurized (i.e. $P = 0$).

Set the control volume.



Continuity equation:

$$0 = \frac{d}{dt} m_{cv} + \sum \dot{m}_o - \sum \dot{m}_\iota$$

Term by Term analysis

Term1: is mass inside the CV change through the time? YES!

$$m_{cv} = \rho \forall$$

$$m_{cv} = \rho (h A_{tank})$$

$$\frac{d}{dt} m_{cv} = \frac{d}{dt}(\rho h A_{tank})$$

$$\frac{d}{dt} m_{cv} = \rho A_{tank} \frac{d}{dt} h$$

Term2: There is one outlet.

$$\dot{m}_o = \rho Q$$

$$\dot{m}_o = \rho V A_{exit}$$

$$\dot{m}_o = \rho A_{exit} \sqrt{\frac{2P}{\rho} + 2gh}$$

Term3: There is no inlet.

Rewrite the continuity equation:

$$\rho A_{tank} \frac{dh}{dt} = -\rho A_{exit} \sqrt{\frac{2P}{\rho} + 2gh}$$

$$\frac{A_{tank}}{A_{exit}} \frac{dh}{\sqrt{\frac{2P}{\rho} + 2gh}} = -dt$$

$$\frac{A_{tank}}{A_{exit}} \int_{h_0}^{0} \frac{dh}{\sqrt{\frac{2P}{\rho} + 2gh}} = -\int_{0}^{t_{empty}} dt$$

$$-\frac{A}{A_e}\frac{1}{g}\left(\frac{2p}{\rho}+2gh\right)^{1/2}\Big|_{h_o}^{0}=\Delta t$$

$$\Delta t=\frac{A}{A_e}\frac{1}{g}\left[\left(\frac{2p}{\rho}+2gh_o\right)^{1/2}-\left(\frac{2p}{\rho}\right)^{1/2}\right]$$

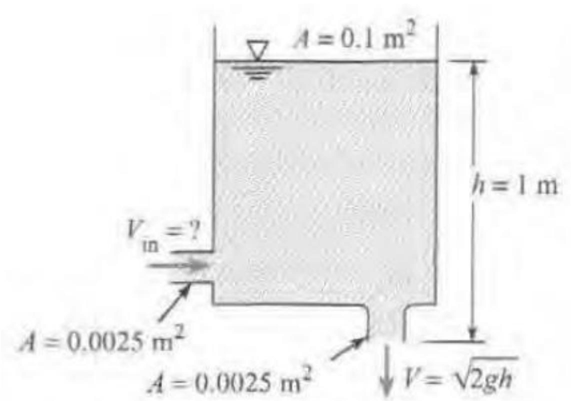
$$\boxed{\Delta t = 329\text{ s or }5.48\text{ min}}$$

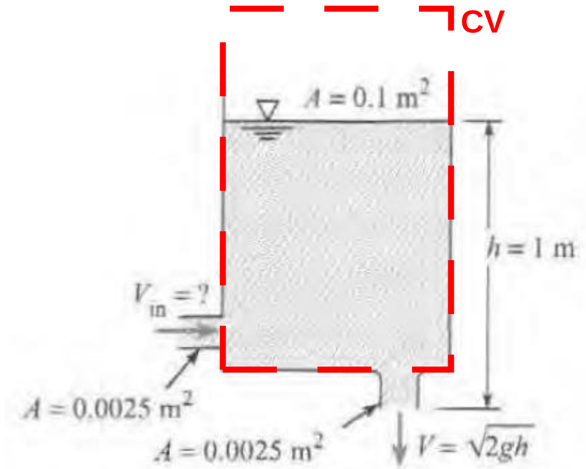
If there is no pressure in tank: $P = 0$

$$\Delta t = \frac{A}{A_e}\sqrt{\frac{2h_o}{g}} = 639\text{ s}$$

$$\boxed{\Delta t = 10.65\text{ min}}$$

4. A tank with the area of $0.1\ m^2$ has a hole in the bottom with a cross-sectional area of $0.0025\ m^2$ and an inlet on the side with a cross-sectional area of $0.0025\ m^2$, as shown. The velocity at the bottom hole is $V = \sqrt{2gh}$, where h is the height of the water above the outlet. At a certain time, the surface level in the tank is $h = 1\ m$ and rising at the rate of $0.1\ \frac{cm}{s}$. Find the velocity at the inlet.





Approach1: Set the control volume:

Continuity equation:

$$0 = \frac{d}{dt}m_{cv} + \sum \dot{m}_o - \sum \dot{m}_i$$

Term by Term analysis

Term1: is mass inside the CV change through the time? YES!

$$m_{cv} = \rho \forall$$

$$m_{cv} = \rho(hA_{tank})$$

$$\frac{d}{dt}m_{cv} = \frac{d}{dt}(\rho h A_{tank})$$

$$\frac{d}{dt}m_{cv} = \rho A_{tank}\frac{dh}{dt}$$

Term2: There is one outlet.

$$\dot{m}_o = \rho Q$$

$$\dot{m}_o = \rho V A_{exit}$$

$$\dot{m}_o = \rho A_{exit}\sqrt{2gh}$$

Term3: There is one inlet.

$$\dot{m}_i = \rho Q$$

$$\dot{m}_i = \rho V_{in} A_{inlet}$$

Rewrite the continuity equation:

$$\rho A_{tank}\frac{dh}{dt} + \rho A_{exit}\sqrt{2gh} - \rho V_{in}A_{inlet} = 0$$

$$\frac{dh}{dt} = 0.001\frac{m}{s}\ Positive\ since\ rising!$$

$$h = 1\ m$$

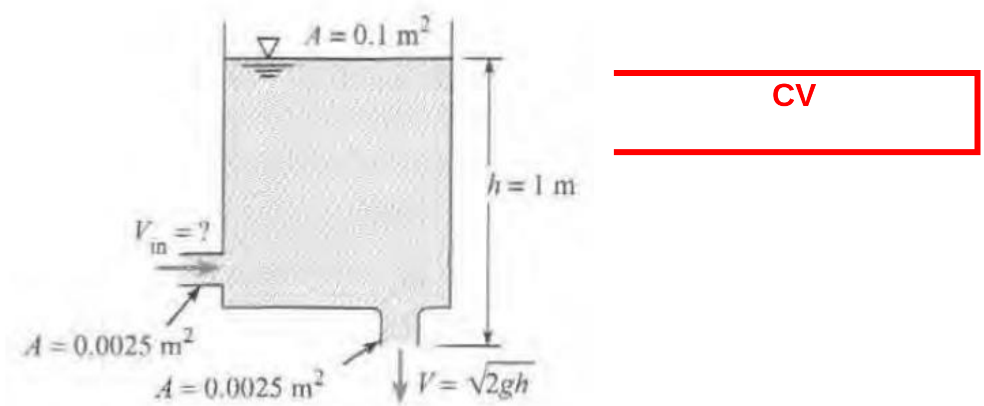
$$-V_{in}A_{in} + V_{out}A_{out} = -A_{tank}(dh/dt)$$

$$-V_{in}(.0025) + \sqrt{2g(1)}(.0025) = -0.1(0.1)\times 10^{-2}$$

$$V_{in} = \frac{\sqrt{19.62}(.0025) + 10^{-4}}{0.0025}$$

$$\boxed{V_{in} = 4.47\ \mathrm{m/s}}$$

Approach2: Set the control volume:



Continuity equation:

$$0 = \frac{d}{dt}m_{cv} + \sum \dot{m}_o - \sum \dot{m}_\iota$$

Term by Term analysis

Term1: is mass inside the CV change through the time? NO! since water level rising in the tank.

$$\frac{d}{dt}m_{cv} = 0$$

Term2: There are **two outlets**. **bottom outlet**:

$$\dot{m}_o = \rho Q$$

$$\dot{m}_o = \rho V A_{exit}$$

$$\dot{m}_o = \rho A_{exit}\sqrt{2gh}$$

top outlet:

$$\dot{m}_o = \rho Q_{rise}$$

$$\dot{m}_o = \rho V_{rise} A_{tank}$$

$$V_{rise} = \frac{dh}{dt}$$

$$\dot{m}_o = \rho A_{tank}\frac{dh}{dt}$$

Term3: There is one inlet.

$$\dot{m}_i = \rho Q$$

$$\dot{m}_i = \rho V_{in} A_{inlet}$$

Rewrite the continuity equation:

$$\rho A_{tank}\frac{dh}{dt} + \rho A_{exit}\sqrt{2gh} - \rho V_{in} A_{inlet} = 0$$

$$\frac{dh}{dt} = 0.001\frac{m}{s} \; Positive\ since\ rising!$$

$$h = 1\ m$$

$$\boxed{V_{in} = 4.47\ \text{m/s}}$$