CVG2116 – Introduction to Fluid Mechanics

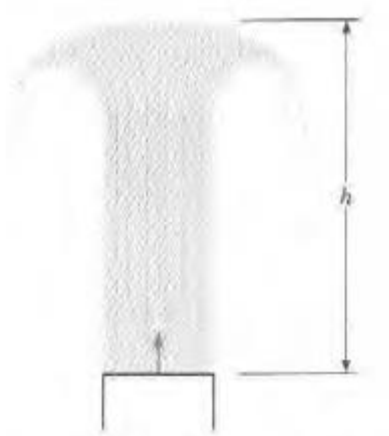
Assignment 2 **Due Date: 02/02/2018**

1. What pressure gradient is required to accelerate water (ρ = 1000 kg/m^3) in a horizontal pipe at a rate of 8m/s^2?

Euler's equation with no change in elevation

$$\frac{\partial p}{\partial s} = -\rho a_s$$
$$= -1,000\,\mathrm{kg/m^3} \times 8\,\mathrm{m/s^2}$$
$$\boxed{\frac{\partial p}{\partial s} = -8,000\ \mathrm{N/m^3}}$$

2. A water jet issues vertically from a nozzle, as shown. The water velocity as it exits the nozzle is 18 m/s. Calculate how high (h) the jet will rise. Hint: Apply the Bernoulli equation along the centerline.



Bernoulli equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

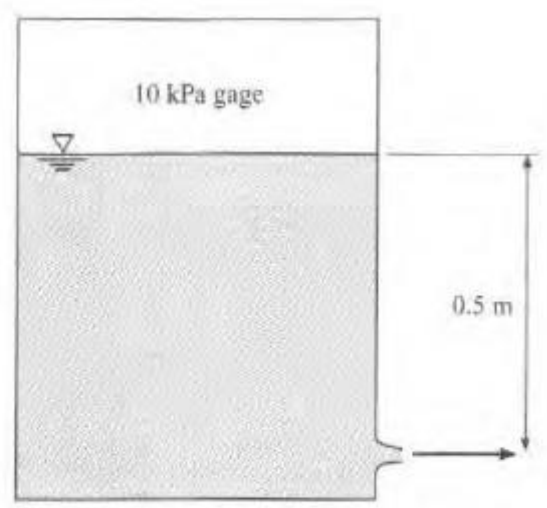
where $p_1 = p_2 = 0$ gage

$$V_1 = 18\frac{m}{s}; V_2 = 0$$

$$0 + \frac{\left(18\frac{m}{s}\right)^2}{2 * 9.81\frac{m}{s^2}} + z_1 = 0 + 0 + z_2$$

$$z_2 - z_1 = h = \frac{324\frac{m^2}{s^2}}{19.62\frac{m}{s^2}} = 16.51m$$

3. A pressure of 10 kPa gage is applied to the surface of water in an enclosed tank. The distance from the water surface to the outlet is 0.5m. The temperature of the water is 20 °C. Find the velocity (m/s) of the water at the outlet. The speed of the water surface is much less than the water speed at the outlet.



Apply the Bernoulli equation between the water surface in the tank (1) and the outlet (2)

$$p_1 + \gamma z_1 + \rho \frac{V_1^2}{2} = p_2 + \gamma z_2 + \rho \frac{V_2^2}{2}$$

Neglect V_1 $(V_1 \ll V_2)$. Also $p_2 = 0$ gage. The Bernoulli equation reduces to

$$\rho \frac{V_2^2}{2} = p_1 + \gamma (z_1 - z_2)$$

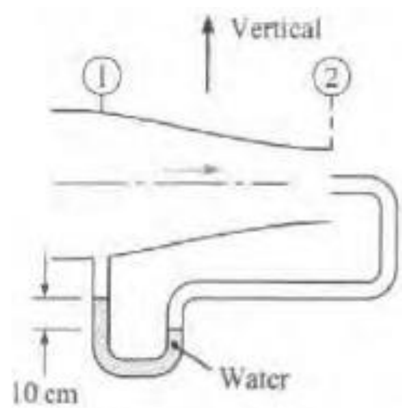
$$V_2 = \sqrt{\frac{2\left(p_1 + \gamma (z_1 - z_2)\right)}{\rho}}$$

Elevation difference $z_1 - z_2 = 0.5$ m. For water at 20°C, $\rho = 998 \text{kg/m}^3$ and $\gamma = 9790$ N/m^3. Therefore

$$V_2 = \sqrt{\frac{2(10,000 \text{ Pa} + 9790 \text{ N/m}^3 (0.5 \text{ m}))}{998 \text{ kg/m}^3}}$$

$$\boxed{V_2 = 5.46 \text{ m/s}}$$

4. The flow-metering device shown consists of a stagnation probe at station 2 and a static pressure tap at station 1. The velocity at station 2 is 1.5 times that at station 1. Air with a density of 1.2 kg/m^3 flows through the duct. A water manometer is connected between the stagnation probe and the pressure tap, and a deflection of 10 cm is measured. What is the velocity at station 2?



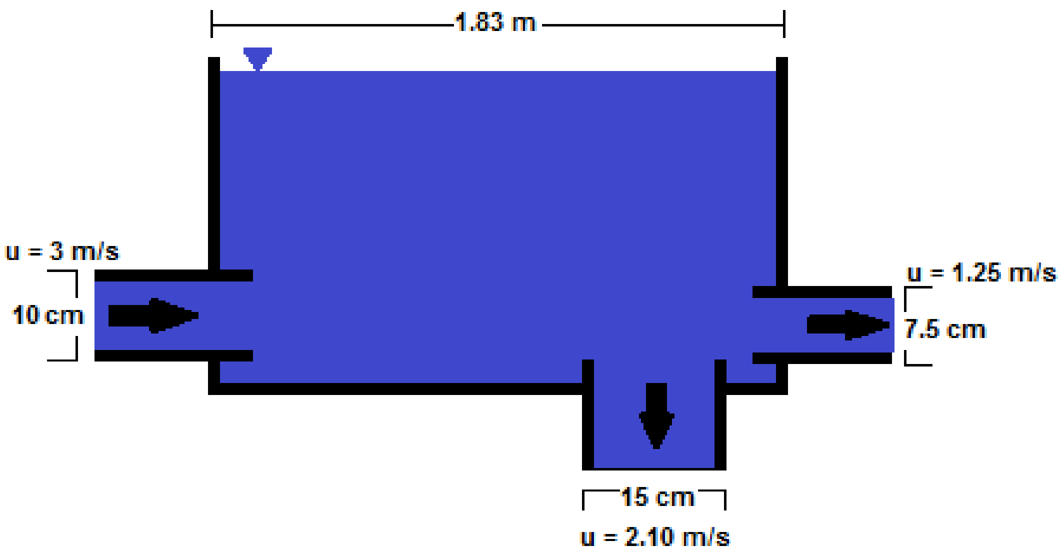
Bernoulli equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} = \frac{p_t}{\gamma}$$

Manometer equation

$$\begin{aligned} p_1 + 0.1 \times 9810 - \overbrace{0.1 \times 1.2 \times 9.81}^{\text{neglect}} &= p_t \\ p_t - p_1 &= 981 \text{ N/m}^2 = \frac{\rho V_1^2}{2} \\ V_1^2 &= \frac{2\,(981\text{ N/m}^2)}{1.2\text{ kg/m}^3} \\ V_1 &= 40.4\text{ m/s} \\ V_2 &= 1.5 V_1 \\ & \boxed{V_2 = 60.7\text{ m/s}} \end{aligned}$$

5. According to the diagram below, does the water level in the tank go up or down? At what rate does the water level in the tank rise or fall?



Solution:

Continuity:

$$\frac{dm_{VC}}{dt} + \sum_{SC} \dot{m_{out}} - \sum_{SC} \dot{m_{in}} = 0 \rightarrow \frac{dm_{VC}}{dt} = \dot{m_{in}} - \sum_{SC} \dot{m_{out}}$$

Mass flow rates:

$$\dot{m_{in}} = \rho * u * A = 1000\frac{kg}{m^3} * 3\frac{m}{s} * \left(\frac{\pi}{4} * (0.1m)^2\right) = 23.56\frac{kg}{s}$$

$$\dot{m_{out}} = \rho_1 * Q_1 + \rho_2 * Q_2$$

$$\dot{m_{out}} = 1000\frac{kg}{m^3} * \left(2.10\frac{m}{s} * \left(\frac{\pi}{4} * (0.15m)^2\right)\right) + 1000\frac{kg}{m^3} * \left(1.25\frac{m}{s} * \left(\frac{\pi}{4} * (0.075m)^2\right)\right)$$

$$\dot{m_{out}} = 37.11\frac{kg}{s} + 5.52\frac{kg}{s} = 42.63\frac{kg}{s}$$

Rate of water accumulation in tank

$$\frac{dm_{VC}}{dt} = 23.56\frac{kg}{s} - 42.63\frac{kg}{s} = -19.07\frac{kg}{s}$$

Rate of change in water level

$$\frac{dh}{dt} = \frac{Rate\ of\ water\ accumultaion}{area}$$

$$\frac{dh}{dt} = \frac{-19.07\,\frac{kg}{s} \Big/ 1000\ kg/m^3}{\left(\frac{\pi}{4} * (1.83m)^2\right)} = -0.00725\,\frac{m}{s}$$