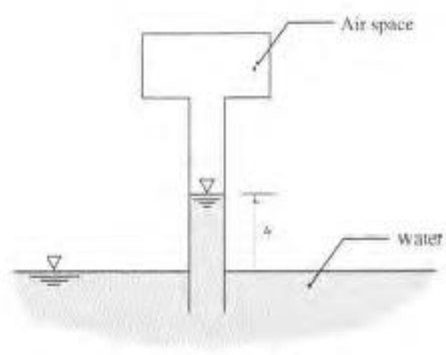
CVG2116 – Introduction to Fluid Mechanics

Assignment 2 **Due Date: 02/02/2018**

1. Imagine two identical tanks. Tank A is filled with water to depth *h*, while Tank B is filled to depth *h* with oil. Which tank has the largest pressure and why? Where in the tank does the largest pressure occur?

   In both tanks, pressure increases with depth, according to $p = -\gamma z$.
   At the bottom of each tank, pressure is given by $p = \gamma h$.
   At the bottom of Tank A, $p = \gamma_{water} h$.
   At the bottom of Tank B, $p = \gamma_{oil} h$.
   Because $\gamma_{oil} < \gamma_{water}$, the pressure in Tank A has the largest pressure.
   The reason is because water has a larger specific weight than oil.
   The largest pressure occurs at the bottom of the tank.

2. As shown below, an air space above a long tube is pressurized to *50 kPa* vacuum. Water (*20* ℃) from a reservoir fills the tube to a height *h*. If the pressure in the air space is changed to *25 kPa* vacuum, will h increase or decrease and by how much? Assume atmospheric pressure is *100 kPa*.



   1. Initial State. Locate point 1 on the reservoir surface; point 2 on the water surface inside the tube:

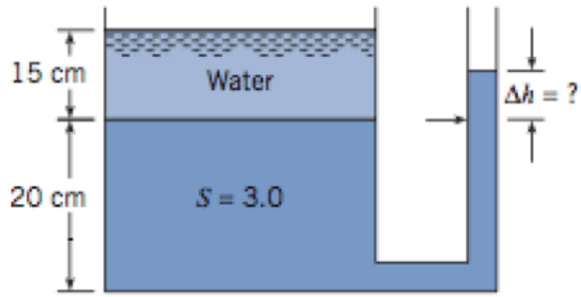
$$\begin{aligned}\frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\ \frac{100\,\text{kPa}}{9790\,\text{N/m}^3} + 0 &= \frac{50\,\text{kPa}}{9790\,\text{N/m}^3} + h \\ h\,(\text{initial state}) &= 5.107\,\text{m}\end{aligned}$$

   2. Final State:

$$\begin{aligned}\frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\ \frac{100\,\text{kPa}}{9790\,\text{N/m}^3} + 0 &= \frac{75\,\text{kPa}}{9790\,\text{N/m}^3} + h \\ h\,(\text{final state}) &= 2.554\,\text{m}\end{aligned}$$

3. A tank is fitted with a manometer on the side, as shown below. The liquid in the bottom of the tank and in the manometer, has a specific gravity (*S*) of *3*. The depth of this bottom liquid is *20 cm*. A *15cm*

layer of water lies on top of the bottom liquid. Find the position of the liquid surface in the manometer (Δh).



1. Hydrostatic equation (location 1 is on the free surface of the water; location 2 is the interface)

$$\begin{aligned}
\frac{p_1}{\gamma_{\text{water}}} + z_1 &= \frac{p_2}{\gamma_{\text{water}}} + z_2 \\
\frac{0\,\text{Pa}}{9810\,\text{N/m}^3} + 0.15\,\text{m} &= \frac{p_2}{9810\,\text{N/m}^3} + 0\,\text{m} \\
p_2 &= (0.15\,\text{m})\left(9810\,\text{N/m}^3\right) \\
&= 1471.5\,\text{Pa}
\end{aligned}$$

2. Hydrostatic equation (manometer fluid; let location 3 be on the free surface)

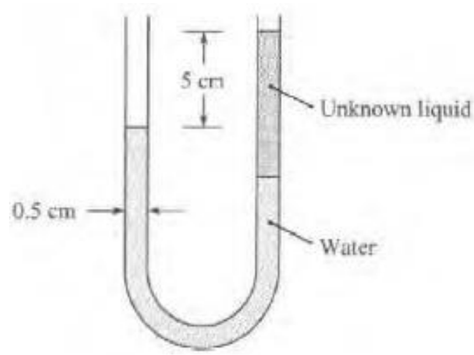
$$\begin{aligned}
\frac{p_2}{\gamma_{\text{man. fluid}}} + z_2 &= \frac{p_3}{\gamma_{\text{man. fluid}}} + z_3 \\
\frac{1471.5\,\text{Pa}}{3\left(9810\,\text{N/m}^3\right)} + 0\,\text{m} &= \frac{0\,\text{Pa}}{\gamma_{\text{man. fluid}}} + \Delta h
\end{aligned}$$

3. Solve for Δh

$$\begin{aligned}
\Delta h &= \frac{1471.5\,\text{Pa}}{3\left(9810\,\text{N/m}^3\right)} \\
&= 0.0500\,\text{m}
\end{aligned}$$

$$\boxed{\Delta h = 5.00\ \text{cm}}$$

4. A device for measuring the specific weight of a liquid consists of a U-tube manometer as shown. The manometer tube has an internal diameter of 0.5cm and originally has water in it. Exactly *2cm³* of unknown liquid is then poured into one leg of the manometer, and a displacement of *5cm* is measured between the surfaces as shown. What if the specific weight of the unknown liquid?



1. Find the length of the column of the unknown liquid.

$$V = (\pi/4)(0.5\,\text{cm})^2\ell = 2\,\text{cm}^3$$

Solve for ℓ

$$\ell = 10.186 \text{ cm}$$

2. Manometer equation (from water surface in left leg to liquid surface in right leg)

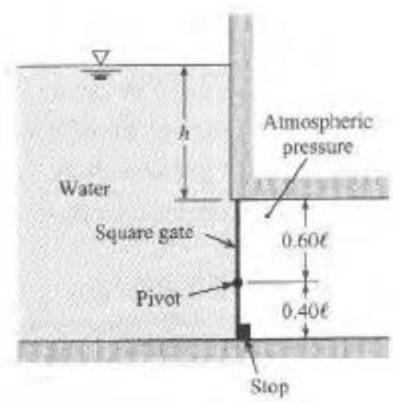
$$0 + (10.186 \text{ cm - } 5 \text{ cm})(10^{-2} \text{ m/cm})(9810 \text{ N/m}^3) - (10.186 \text{ cm})(10^{-2} \text{ m/cm})\gamma_{\text{liq.}} = 0$$

Solve for $\gamma_{\text{liq.}}$

$$508.7 \text{ Pa } - 0.10186\gamma_{\text{liq.}} = 0$$

$$\boxed{\gamma_{\text{liq.}} = 4995 \text{ N/m}^3}$$

5. The square gate shown is eccentrically pivoted so that it automatically opens at a certain value of *h*. What is that value in terms of *l*?



Center of pressure (when the gate opens)

$$\begin{aligned} y_{cp} - \bar{y} &= 0.60\ell - 0.5\ell \\ &= 0.10\ell \end{aligned} \quad (1)$$

Center of pressure (formula)

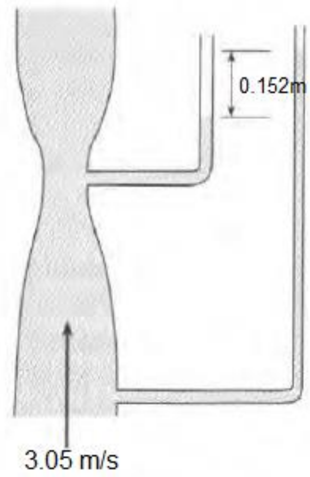
$$\begin{aligned} y_{cp} - \bar{y} &= \frac{\bar{I}}{\bar{y}A} \\ &= \frac{(\ell \times \ell^3)/12}{(h + \ell/2)\ell^2} \end{aligned} \quad (2)$$

Combine Eqs. (1) and (2)

$$\begin{aligned} 0.10\ell &= \frac{(\ell \times \ell^3)/12}{(h + \ell/2)\ell^2} \\ 0.10 &= \frac{\ell}{12(h + \ell/2)} \\ h &= \frac{5}{6}\ell - \frac{1}{2}\ell \\ &= \frac{1}{3}\ell \end{aligned}$$

$$\boxed{h = \ell/3}$$

6. Water (20 °C) flows through a vertical contraction (venturi) section. Piezometers are attached to the upstream pipe and minimum area section as shown. The velocity in the pipe is *3.05 m/s*. The difference in elevation between the two water levels in the piezometers is *0.152m*. What is the velocity at the minimum area?



Apply the Bernoulli equation between the pipe (1) and the minimum area (2)

$$p_1 + \gamma z_1 + \rho \frac{V_1^2}{2} = p_2 + \gamma z_2 + \rho \frac{V_2^2}{2}$$

From problem statement, $V_1 = 10$ ft/s. Rewriting equation

$$\rho \frac{V_2^2}{2} = \rho \frac{V_1^2}{2} + (p_1 + \gamma z_1) - (p_2 + \gamma z_2)$$

The difference in the elevation in piezometers gives the change in piezometric pressure, $(p_1 + \gamma z_1) - (p_2 + \gamma z_2) = \gamma \Delta h$ so

$$V_2 = \sqrt{V_1^2 + \frac{2\gamma \Delta h}{\rho}} = \sqrt{V_1^2 + 2g\Delta h}$$

$$V_2 = \sqrt{\left(3.05\ \frac{m}{s}\right)^2 + 2 * 9.81 \frac{m}{s^2} * 0.152m} = 3.5 \frac{m}{s}$$

7. The velocity components for a two-dimensional flow are $u = \frac{Cx}{(x^2+y^2)}$ and $v = \frac{Cy}{(x^2+y^2)}$ where C is a constant. Is the flow irrotational?

ANALYSIS

Apply equations for flow rotation in $x - y$ plane.

$$\begin{aligned}\partial v/\partial x - \partial u/\partial y &= (2Cy/(y^2+x^2)^2) - (2C(y^2-x^2)2y/(y^2+x^2)^3) \\ &\quad +(2Cy/(y^2+x^2)^2) - (4Cxy(2x)/(y^2+x^2)^3) \\ &= 0 \quad \boxed{\text{The flow is irrotational}}\end{aligned}$$