MATH2004 Test #2 Answers

1. $f(x,y) = x^3 + y^3 - 3x - 12y$ $f_{xx} = 6x$, $f_{yy} = 6y$, $f_{xy} = 0$. ∴ D(1, -2) < 0 and D(1, 2) > 0. These give (c) S.P. & min.

2. If $w = f(x,y,z)$ where $x = st^2$, $y = s^2 - t^3$, $z = -st$ then the partial derivative $w_s = f_x x_s + f_y y_s + f_z z_s = t^2 f_x + 2sf_y - tf_z$ so the answer is (c).

3. $\nabla f = (2xy+yz \quad x^2+xz \quad xy+2z)$ so $\nabla f\,(2, 1, -1) = (3\ 2\ 0)$ and $|\nabla f|^2 = 9+4$ so (b) $\sqrt{13}$.

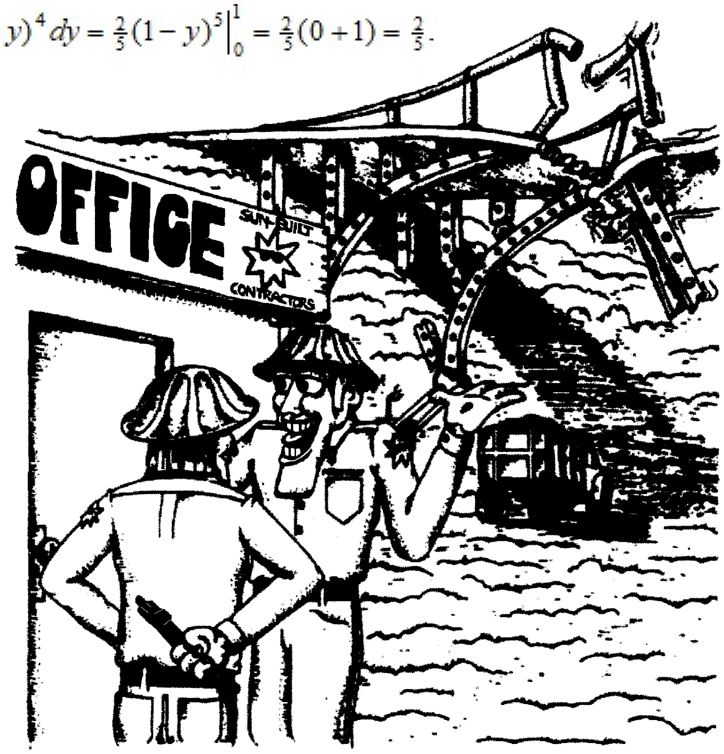
4. **F** = (P,Q,R) gives $P_y = z+1$, $Q_x = z+b$; $P_z = y+a$, $R_x = cy-1$; $Q_z = x$, $R_y = cx$ so $b = c = 1$ and $a = -1$ so (d) (-1 1 1).

5. $$\int_{-1}^{1}\int_{0}^{2} 6x^2 y\,dy\,dx = \int_{-1}^{1} y^2\Big|_0^2 3x^2\,dx = \int_{-1}^{1} 12x^2\,dx = 4x^3\Big|_{-1}^{1} = 8. \quad (c).$$

6. The circle is $x^2 + y^2 = 4$ so for C take $x = 2\cos t$, $y = 2\sin t$, $t \in [0, \pi/4]$.
Then $x' = -2\sin t$, $y' = 2\cos t$ so $x'^2 + y'^2 = 4$.
Thus $\int_C x - y\, ds = \int_0^{\pi/4} (2\cos t - 2\sin t)2\,dt = 4(\sin t + \cos t)\Big|_0^{\pi/4} = 4(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}) = 4\sqrt{2} - 1).$

7. For $0 \le y \le 1$, we get $y - 1 \le x \le 1 - y$ as the region $\Re$.
Then $\int_{\Re}\int 3x^2(1 - y)\, dA =$
$$\int_0^1\int_{y-1}^{1-y} 3x^2(1-y)\,dx\,dy = \int_0^1 x^3\Big|_{y-1}^{1-y}(1-y)\,dy = \int_0^1 2(1-y)^4\,dy = \tfrac{2}{5}(1-y)^5\Big|_0^1 = \tfrac{2}{5}(0+1) = \tfrac{2}{5}.$$

8. $f(x,y) = x^2 + y^2$.
Let $g(x,y) = x^4 + 4y^4 - 4$ and $F = f - \lambda g$.
$F_x = 0$ gives $2x = 4\lambda x^3$ so
$x = 0$ or $2\lambda x^2 = 1$.
$F_y = 0$ gives $2y = 16\lambda y^3$ so
$y = 0$ or $8\lambda y^2 = 1$.
$x = 0 = g$ implies $y^4 = 1$ so $y = \pm 1$.
$(0,\pm 1)$ $f(x,y) = 1$.
$y = 0 = g$ implies $x^4 = 4$ so $x = \pm\sqrt{2}$.
$(\pm\sqrt{2},0)$ $f(x,y) = 2$.
Otherwise $x^2 = 4y^2$ so:
$g = 0$ gives $20y^4 = 4$.
Thus $y^2 = 1/\sqrt{5}$ and $x^2 = 4/\sqrt{5}$ (4 C.P.)
$f(x,y) = 4/\sqrt{5} + 1/\sqrt{5} = 5/\sqrt{5} = \sqrt{5}$.
Hence: minimum is 1
maximum is 2.



But boss, I just left out a decimal point. Don't I get at least partial credit?