<u>MATH2004</u> <u>Test #1</u> <u>Answers</u>

1. $|v| = 9$, $|w| = 5\sqrt{2}$, $v.w = 45$ so the cosine is $1/\sqrt{2}$. (c)
2. normal vector $\underline{n}$ to $4 + 3x - y - 3z = 0$ is (3 -1 -3) (b)
3. (1 2 -1)×(-2 1 1) = (3 1 5) (d)
4. $r^2 = 4 + 4$. (-2 -2) is in the 4th quadrant (e)
5. $\nabla f(x,y,z) = e^{xy^2z^3}(y^2z^3 + 2xyz^3 + 3xy^2z^2)$ (d)
6. v = (-2 3) - (1 -1) = (-3 4) so unit vector is $(1/5)v$, ∇f = (-2 1). (a)
7. $x' = -3\cos^2 t \sin t$, $y' = 3\sin^2 t \cos t$, $x'^2 + y'^2 = 9\cos^2 t\sin^2 t(\cos^2 t + \sin^2 t) = 9\cos^2 t \sin^2 t$.

$$\text{Length} = \int_0^{\pi/4} 3\cos t \sin t\,dt = \tfrac{3}{2}\sin^2 t\Big|_0^{\pi/4} = \tfrac{3}{2}\left(\tfrac{1}{2} - 0\right) = \tfrac{3}{4}.$$

8. At $t = -1$, $(x\ y) = (0\ 0)$, then $x > 0$, $y = t^3(t^2 - 1) > 0$ so start in 1st quadrant. When $t = 0$, $(x\ y) = (1\ 0)$ and $t > 0$ gives $x < 1$, $y < 0$, 4th quadrant. Hence the curve goes clockwise. Area A (= $\int y\,dx$) is

$$A = \int_{-1}^{1} (t^5 - t^3)(-4t^3)\,dt = \int_{-1}^{1} 4t^6 - 4t^8\,dt = \tfrac{4}{7}t^7 - \tfrac{4}{9}t^9\Big|_{-1}^{1} = \tfrac{16}{63}.$$

9. The vector from (-1 0 2 0) to (3 -2 4 -1) is v = (4 -2 2 -1) so $|v| = 5$ and the unit vector in this direction is u = (1/5)(4 -2 2 -1).
$\nabla f(w,x,y,z) = (2we^x \quad w^2e^x \quad 3y^2e^{3z} \quad 3y^3e^{3z})$ so ∇f(-1 0 2 0) = (-2 1 12 24).
Then $D_u f = \nabla f . u$ = (1/5)(-8 -2 24 -24) = -2.