Actu 257 Final Solutions

1.

$${}_tV(\bar{A}_x) \overset{\underline{prosp.}}{=} \bar{A}_{x+t} - P(\bar{A}_x)\ddot{a}_{x+t}$$

$$= \bar{A}_{x+t} - P(\bar{A}_x)\ddot{a}_{x+t} - \frac{\bar{A}_x - P(\bar{A}_x)\ddot{a}_x}{{}_tE_x} =$$

$$= \bar{A}_{x+t} - \frac{\bar{A}_x}{{}_tE_x} - P(\bar{A}_x)\cdot\ddot{a}_{x+t} + P(\bar{A}_x)\frac{\ddot{a}_x}{{}_tE_x} =$$

$$= P(\bar{A}_x)\frac{\ddot{a}_x - {}_tE_x\ddot{a}_{x+t}}{{}_tE_x} - \frac{\bar{A}_x - {}_tE_x\bar{A}_{x+t}}{{}_tE_x} =$$

$$= P(\bar{A}_x)\frac{\ddot{a}_{x:\overline{t}|}}{{}_tE_x} - \frac{\bar{A}^{\,1}_{x:\overline{t}|}}{{}_tE_x} \overset{retrosp.}{=} {}_tV(\bar{A}_x)$$

2. $$(\bar{I}\bar{A})_x = \int_0^{\omega-x} t v^t {}_tp_x \mu_{x+t}\,dt = E[T v^T] \overset{T=K+S}{=}$$

$$= E[(K+1+S-1)v^T] = E[(K+1)v^T] + E[(S-1)v^T]$$

$$= (I\bar{A})_x + E[(S-1)v^{K+1}v^{S-1}]$$

$$(I\bar{A})_x = \sum_{k=0}^{\omega-x-1} {}_{k|}\bar{A}_x \overset{UDD}{=} \sum_{k=0}^{\omega-x-1} \frac{i}{\delta}\,{}_{k|}A_x = \frac{i}{\delta}(IA)_x$$

$$E[(S-1)v^{K+1}\cdot v^{S-1}] \underset{\substack{K+1 \,\&\, S-1\\ \text{indep.}}}{\overset{UDD}{=}} E[v^{K+1}]\cdot E[(S-1)v^{S-1}]$$

$$= A_x \cdot E[(S-1)v^{S-1}]$$

Now: $E[(S-1)v^{S-1}] \xlongequal[S\sim U(0,1)]{UDD} \int_0^1 (s-1)v^{s-1}ds =$

$$= -\int_0^1 (1-s)(1+i)^{1-s}\,ds \xlongequal[\substack{\tau=1-s\\ d\tau=-ds\\ 0\le s\le 1\\ 1\le\tau\le 0}]{} -\int_0^1 \tau(1+i)^{\tau}\,d\tau = \circledast$$

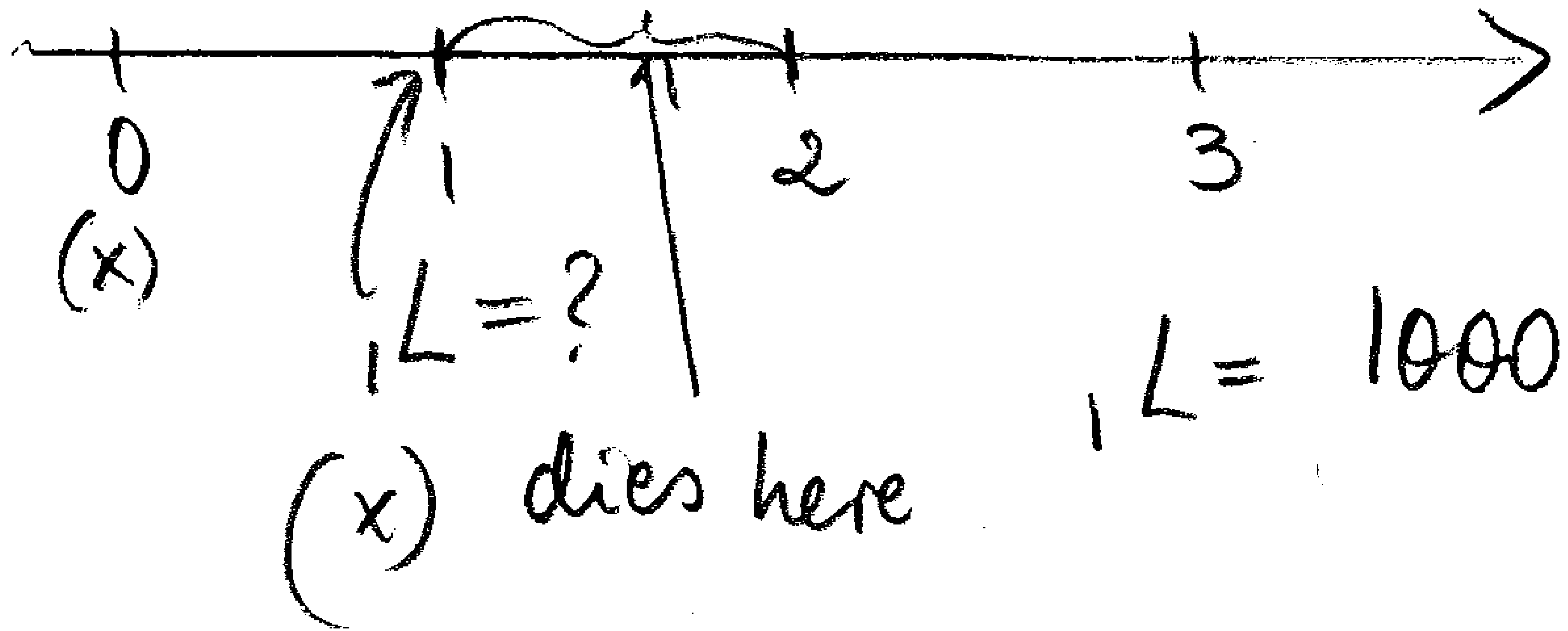
$$\left\{\begin{matrix} u=\tau & dv=e^{\tau\delta}d\tau \\ du=d\tau & v=\frac{1}{\delta}e^{\tau\delta}\end{matrix}\right\} \circledast = -\left[\frac{1}{\delta}\tau e^{\tau\delta}\Big|_0^1 - \frac{1}{\delta}\int_0^1 e^{\tau\delta}d\tau\right] =$$

$$= -\left[\frac{e^{\delta}}{\delta} - \frac{1}{\delta^2}\left[e^{\tau\delta}\right]_0^1\right] = -\left[\frac{1+i}{\delta} - \frac{1+i-1}{\delta^2}\right] =$$

$$= -\left[\frac{1+i}{\delta} - \frac{i}{\delta^2}\right]$$

So $(\bar{I}\bar{A})_x = \frac{i}{\delta}(IA)_x - \left[\frac{1+i}{\delta} - \frac{i}{\delta^2}\right]\cdot A_x$

3.



${}_1L = 1000v - P = ?$

$$P = \frac{1,000\,A_{x:\overline{3|}}}{\ddot{a}_{x:\overline{3|}}} = 1,000\ \frac{1-d\,\ddot{a}_{x:\overline{3|}}}{\ddot{a}_{x:\overline{3|}}} = 279.21179$$

So ${}_1L = 1,000(1.1)^{-1} - P = 629.879 \simeq \underline{\underline{629.88}}$

4.

$$0.8\, q_{[60]+0.4} = \frac{\ell_{[60]+0.4} - \ell_{[60]+1.2}}{\ell_{[60]+0.4}}$$

$$\ell_{[60]+0.4} = 0.6 \cdot \ell_{[60]} + 0.4 \cdot \ell_{[60]+1} =$$

$$= 0.6 \cdot 80{,}625 + 0.4 \cdot 79{,}954 = 80{,}356.6$$

$$\ell_{[60]+1.2} = 0.8\, \ell_{[60]+1} + 0.2 \cdot \ell_{62} =$$

$$= 0.8 \cdot 79{,}954 + 0.2 \cdot 78{,}839 = 79{,}731$$

$$0.8\, q_{[60]+0.4} = \underline{\underline{0.00778529\text{7}}}$$

5. $$E[L^*] = 1{,}000 A_{60}^{correct} - P^{wrong} \cdot \ddot{a}_{60}^{correct}$$

$$A_{60}^{wrong} = v q_{60}^{wrong} + v p_{60}^{wrong} \cdot A_{61} \Rightarrow v A_{61} = \frac{A_{60}^{w} - v q_{60}^{w}}{p_{60}^{w}}$$

$$A_{60}^{correct} = v\, q_{60}^{corr.} + v p_{60}^{corr.} \cdot A_{61} = v\, q_{60}^{cor} + p_{60}^{cor} \cdot \frac{A_{60}^{w} - v q_{60}^{w}}{p_{60}^{w}} =$$

$$= 0.01\,(1.06)^{-1} + 0.99\, \frac{0.40554 - 0.1\,(1.06)^{-1}}{0.9} = 0.351754377$$

$$P^{wrong} = \frac{1{,}000\, A_{60}^{wrong}}{\ddot{a}_{60}^{wrong}} = \frac{405.54\, d}{1 - 0.40554} = 38.61503607$$

$$\simeq 38.62$$

$$E[L^*] = 351.754 - 38.62 \cdot \frac{1 - 0.351754}{0.06} \cdot 1.06 =$$

$$= -90.478 \simeq -\underline{\underline{90.48}}$$

6.

$$A_{40} = vq_{40} + vp_{40}A_{41}$$

$$p_{40} = \frac{s(41)}{s(40)} = 0,95 \Rightarrow q_{40} = 0.05$$

$$A_{41} \overset{UDD}{=} \frac{\delta}{i}\bar{A}_{41} = \frac{\ln(1.06)}{0.06}\cdot 0.54 = 0,524420173$$

So $A_{40} = \frac{0,05}{1.06} + \frac{0,95}{1.06}\cdot A_{41} = 0,517169\ldots$ Ⓑ

7.

$$\bar{P}(\bar{A}_x) = \frac{\bar{A}_x}{\bar{a}_x} = \mu = 0.03$$

$${}^2\bar{A}_x = \frac{\mu}{\mu+2\delta} = \underbrace{\frac{0.03}{0.03+2\delta} = 0.2} \Rightarrow 0.03 = 0.006 + 0.4\delta$$

$$0,4\delta = 0.024$$

$$\delta = 0.06$$

$$Var(L) = \left(1 + \frac{\bar{P}(\bar{A}_x)}{\delta}\right)^2\left({}^2\bar{A}_x - \bar{A}_x^2\right) =$$

$$= \left(1 + \frac{0.03}{0.06}\right)^2\left\{0.2 - \underset{\substack{\| \\ \left(\frac{1}{3}\right)^2}}{\left(\frac{0.03}{0.03+0.06}\right)^2}\right\} = \frac{9}{4}\left[\frac{2}{10} - \frac{1}{9}\right] =$$

$$= \frac{\overset{1}{\cancel{9}}}{4}\,\frac{18-10}{\underset{10}{\cancel{90}}} = \underline{\underline{0.2}}$$

Ⓐ

8. $\mathring{e}_{\omega-1} \overset{UDD}{=} e_{\omega-1} + \frac{1}{2}$ if there is no UDD $\underbrace{e_{\omega-1} + \frac{1}{2}}$ approximates $\mathring{e}_{\omega-1}$

$e_{\omega-1} = p_{\omega-1} = 0$ independently of the formula for l_x

So $\mathring{e}_{\omega-1} \approx 1/2$ UDD approximation

$$\mathring{e}_{\omega-1} \overset{exact}{=} \int_0^1 {}_tp_{\omega-1}\,dt = \int_0^1 \frac{l_{\omega-1+t}}{l_{\omega-1}}\,dt = \int_0^1 (1-t)^6\,dt \underset{dr=-dt}{\overset{r=1-t}{=}}$$

$$= \int_0^1 r^6\,dt = \frac{1}{7}r^7\Big|_0^1 = \frac{1}{7} \qquad \frac{1}{2} - \frac{1}{7} = \frac{7-2}{14} = 5/14$$

Ⓔ

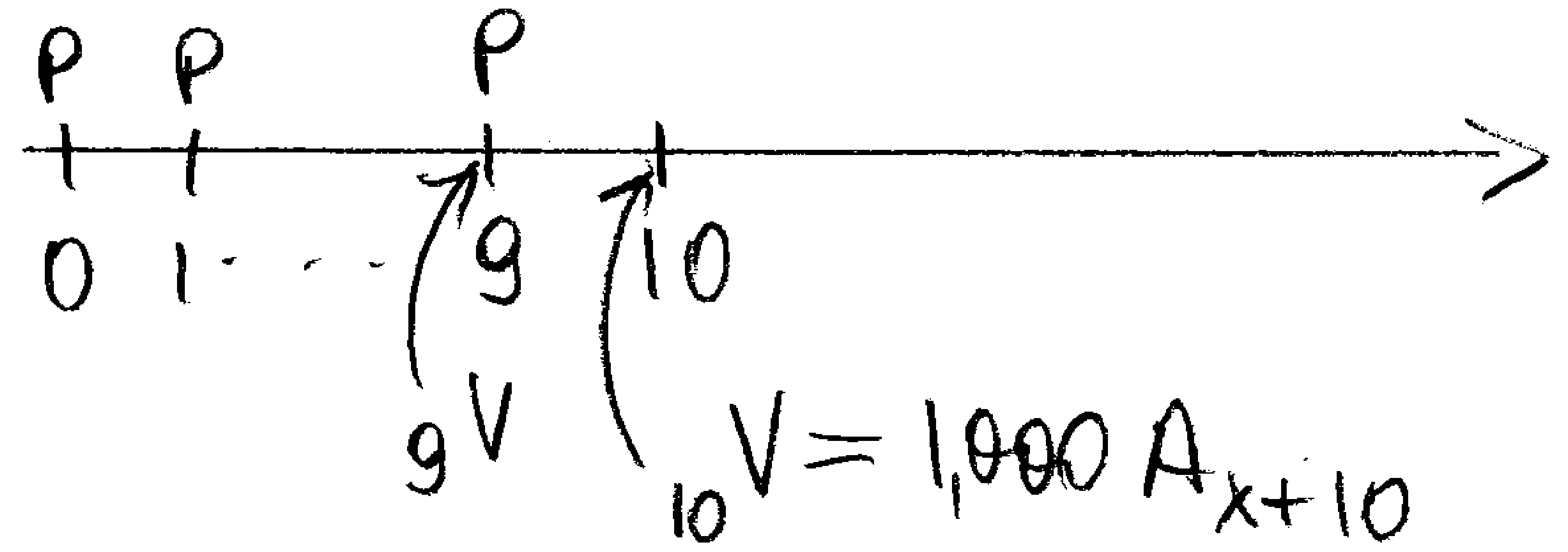
9. $$\mathrm{Var}(L) = \left(1+\frac{P_{x:\overline{n|}}}{d}\right)^2\left({}^2A_{x:\overline{n|}} - \underset{\substack{\uparrow\\ \text{we need this}}}{A_{x:\overline{n|}}^2}\right)$$

$$P_{x:\overline{n|}} = \frac{A_{x:\overline{n|}}}{\ddot{a}_{x:\overline{n|}}} = \frac{A_{x:\overline{n|}}}{1-A_{x:\overline{n|}}}\cdot d \Rightarrow \frac{P_{x:\overline{n|}}}{d}\left(1-A_{x:\overline{n|}}\right) = A_{x:\overline{n|}}$$

Hence $$A_{x:\overline{n|}} = \frac{P_{x:\overline{n|}}}{d}\cdot\frac{1}{1+\frac{P_{x:\overline{n|}}}{d}} = 0.585\cdot\frac{1}{1.585} = 0.36908$$

$$\mathrm{Var}(L) = (1.585)^2\left(0.1774-(0.36908)^2\right) = \underline{\underline{0.10343715}}$$

Ⓑ

10. 

$$\left({}_9V+P\right)(1+i) = 1{,}000\;q_{x+9} + p_{x+9}\cdot{}_{10}V$$

$$(322.87+32.88)(1.06) = 12.62 + 0.98738\cdot{}_{10}V$$

$$\Rightarrow {}_{10}V = \frac{377.095-12.62}{0.98738} = 369.1334643$$

$$1{,}000\,P_{x+10} = \frac{1{,}000\,A_{x+10}\cdot d}{1-A_{x+10}} = \frac{{}_{10}V\cdot d}{1-0.3691334643} = \underline{\underline{33.12}}$$

Ⓔ

11. $\mathring{e}_{x:\overline{n|}} \overset{UDD}{=} e_{x:\overline{n|}} + \frac{1}{2}$ is not true

$$e_{60:\overline{1.5|}} = \int_0^1 {}_tp_{60}\,dt + p_{60}\int_0^{1/2} {}_tp_{61}\,dt \overset{UDD}{=} \int_0^1 (1-t\cdot q_{60})\,dt +$$
$$+\,0.98\cdot\int_0^{1/2}(1-t\cdot q_{61})\,dt = 1-\tfrac{1}{2}\cdot q_{60} + 0.98\left[0.5-\tfrac{1}{8}\cdot q_{61}\right] =$$
$$= 1-0.01+0.98(0.5-0.00275) = 1.477305 \simeq 1.48$$ Ⓓ

12. $$P_{30:\overline{15|}} - {}_{15}P_{30} = \frac{A_{30:\overline{15|}}}{\ddot{a}_{30:\overline{15|}}} - \frac{A_{30}}{\ddot{a}_{30:\overline{15|}}}$$

$$v^{15}(1-{}_{15}q_{30})(d + P_{30:\overline{15|}}) = v^{15}\cdot{}_{15}p_{30}\left(d + \frac{A_{30:\overline{15|}}}{\ddot{a}_{30:\overline{15|}}}\right) =$$

$$= {}_{15}E_{30}\;\frac{d\,\ddot{a}_{30:\overline{15|}} + A_{30:\overline{15|}}}{\ddot{a}_{30:\overline{15|}}} = \frac{{}_{15}E_{30}}{\ddot{a}_{30:\overline{15|}}}$$

So Everything $= 1 - \frac{(A_{30:\overline{15|}} - A_{30})}{\cancel{\ddot{a}_{30:\overline{15|}}}}\cdot\frac{\cancel{\ddot{a}_{30:\overline{15|}}}}{{}_{15}E_{30}} =$

$$= \frac{{}_{15}E_x - A_{30:\overline{15|}} + A_{30}}{{}_{15}E_{30}} = \frac{-A^{\;1}_{30:\overline{15|}} + A_{30}}{{}_{15}E_x} =$$

$$= \frac{{}_{15}E_x A_{45}}{{}_{15}E_x} = A_{45}$$ Ⓔ

13. $$\ddot{a}_{x:\overline{n|}} = \sum_{k=0}^{n-1} v^k\,{}_kp_x \underset{\uparrow}{\geq} {}_np_x \sum_{k=0}^{n-1} v^k = {}_np_x\cdot\ddot{a}_{\overline{n|}}$$

${}_kp_x \geq {}_np_x$ for $k \leq n$

So I $\geq$ II

$$\frac{\ddot{a}_{x:\overline{n|}}}{v^n\,{}_np_x} \geq \frac{\cancel{{}_np_x}\cdot\ddot{a}_{\overline{n|}}}{v^n\,\cancel{{}_np_x}} = \ddot{a}_{\overline{n|}}(1+i)^n = \ddot{s}_{\overline{n|}}$$

so III $\geq$ IV

$$\ddot{s}_{\overline{n|}} = (1+i)^n\,\ddot{a}_{\overline{n|}} = (1+i)^n\sum_{k=0}^{n-1} v^k \geq \sum_{k=0}^{n-1} v^k \underset{{}_kp_x \leq 1}{\geq} \sum v^k\,{}_kp_x = \ddot{a}_{x:\overline{n|}}$$

so IV $\geq$ I & II $\leq$ I $\leq$ IV $\leq$ III Ⓓ

14. $$P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} = \frac{1-d\ddot{a}_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} = \frac{1}{\ddot{a}_{x:\overline{n}|}} - d$$

$$\Rightarrow \ddot{a}_{x:\overline{n}|} = \frac{1}{P_{x:\overline{n}|}+d} = \frac{1}{0.0455+\frac{0.1}{1.1}} = 7.330889$$

$$\ddot{a}^{(4)}_{x:\overline{n}|} = \ddot{a}^{(4)}_{x} - {}_{n}E_{x}\cdot \ddot{a}^{(4)}_{x+n} \overset{UDD}{=} \alpha(4)\ddot{a}_{x} - \beta(4) -$$

$$-{}_{n}E_{x}\left[\alpha(4)\ddot{a}_{x+n} - \beta(4)\right] = \alpha(4)\ddot{a}_{x:\overline{n}|} - \beta(4)(1-{}_{n}E_{x})$$

$$\underset{\text{plug in values}}{=} 7.1708897 \simeq 7.17 \quad Ⓓ$$

15. $$\mathring{e}_{20:\overline{20}|} = \int_0^{20} {}_{t}p_{20}\,dt = \int_0^{20} \frac{l_{20+t}}{l_{20}}\,dt = \int_0^{20} \frac{l_{20}-t\cdot k}{l_{20}}\,dt$$

$$= \left[20 - \frac{1}{2}\left[t^2\right]\Big|_0^{20}\cdot\frac{k}{l_{20}}\right] = 18$$

$$20 - 200\frac{k}{l_{20}} = 18 \Rightarrow \frac{k}{l_{20}} = 0.01$$

$${}_{30|10}q_{30} = {}_{40}q_{30} - {}_{30}q_{30} = \frac{l_{60}-l_{70}}{l_{30}}$$

$$l_{30} = l_{20} - 10\cdot k \qquad l_{60} = l_{20} - 40k \qquad l_{70} = l_{20} - 50k$$

So $${}_{30|10}q_{30} = \frac{l_{20}-40k-l_{20}+50k}{l_{20}-10k} = \frac{10k \;\;/:l_{20}}{l_{20}-10k \;\;/:l_{20}} =$$

$$= \frac{10\frac{k}{l_{20}}}{1-10\frac{k}{l_{20}}} = \frac{10\cdot 0.01}{1-10\cdot 0.01} = \frac{0.1}{0.9} = \frac{1}{9}$$