University of Calgary

Schulich School of Engineering

Engineering Dynamics

ENGG 349 (Spring 2017)

Instructor: Dr. Ahmad Ghasemloonia

Midterm Examination

Friday, 9 June 2017, 13:00 – 15:00

Student Name:

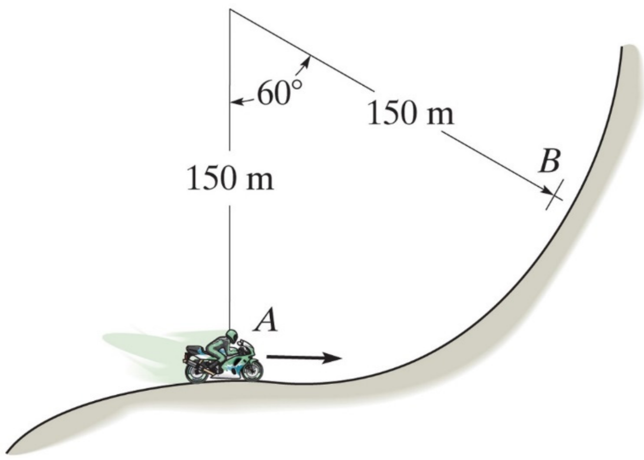
Seating list#

Instructions:

- ✓ Please write your name and student ID on each page of the exam booklet.
- ✓ Please write all answers on the space provided in each question.
- ✓ Please show all significant steps clearly in all problems.
- ✓ Please draw a box around your final answers in each part of the questions.
- ✓ The duration of this exam is 2 hours.
- ✓ 55 is the full mark for this examination.
- ✓ Attempt all 4 questions.
- ✓ Only non-programmable calculators may be used during this test.
- ✓ A formula sheet is attached to the end of the exam booklet.

<u>Problem 1: (10 Marks)</u>, *Dynamics, Hibbeler, 14th ed., Problem 12-138*

The motorcycle is traveling along a circular path at $40\ \frac{m}{s}$ when it is at point A. If the speed is then decreased at $a_t = -(0.05\ s)\ \frac{m}{s^2}$, where "s" is in meters measured from point A, determine its magnitude of the velocity and magnitude of the acceleration when the motorcycle reaches point B.



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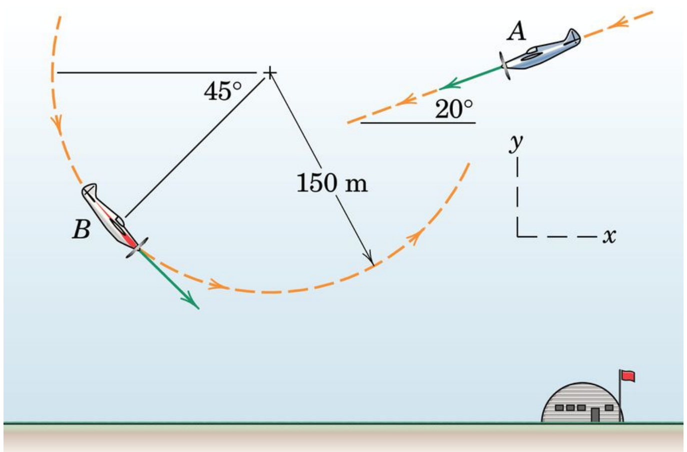
Problem 2: (10 Marks), *Dynamics, Meriam, 8th ed., Problem 2-234 modified*

Two airplanes are performing at an air show. Plane A travels along the path shown and has a velocity of $125\ \frac{m}{s}$ which is increasing at a rate of $8\ \frac{m}{s^2}$. Meanwhile, plane B travels along the circular loop shown at a constant speed of $150\ \frac{m}{s}$. Determine the magnitude of relative velocity and magnitude of relative acceleration of plane B with respect to plane A at the instant shown.



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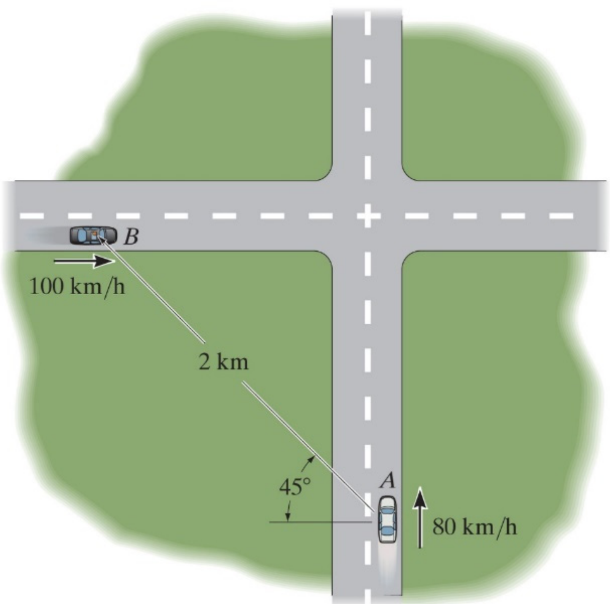
<u>Problem 3: (15 Marks)</u>, *Dynamics, Hibbeler, 14th ed., Problem F12-45 modified*

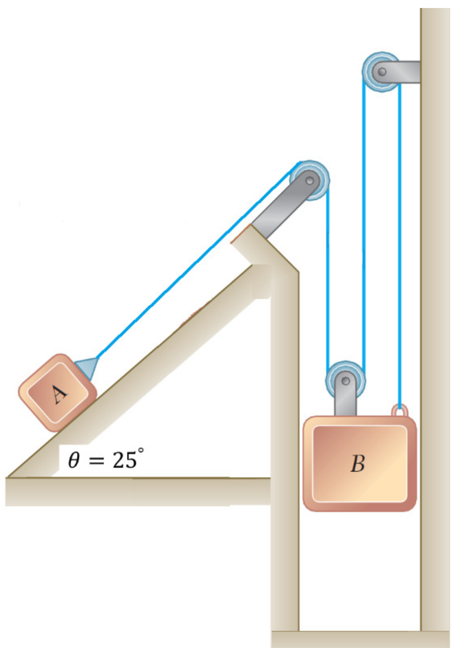
Car B is traveling due east at $100\ \frac{km}{h}$ and is being tracked by a radar in police car A. Police car A is traveling with a constant velocity of $80\ \frac{km}{h}$ due north. Since car B is traveling above the speed limit, the driver pushes his brake pedal and decelerates at the rate of $1000\ \frac{km}{h^2}$ to avoid getting a speed ticket. The radar in the police car tracks car B in the polar coordinate system. Determine the values of $\dot{r}$, $\dot{\theta}$, $\ddot{r}$ and $\ddot{\theta}$ of car B as measured by the radar in the police car A at this instant.



Problem 4: (20 Marks), *Dynamics, Beer, 11th ed., Problem 11-186 modified*

Determine the acceleration of block B, acceleration of block A, and the tension in the cable if the system is released from rest. The coefficient of static and kinetic friction between block A and the wedge surface are $\mu_s = 0.2$ and $\mu_k = 0.15$, respectively. Neglect masses of the pulleys and associated friction in the pulleys. Consider $m_A = 15\ kg$ and $m_B = 8\ kg$.

FBDs and kinetic diagrams for blocks A and B is mandatory.



Position, Velocity and Acceleration

$$v = \dot{x} = \frac{dx}{dt}$$

$$a = \dot{v} = \ddot{x} = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v\frac{dv}{dx}$$

Constant acceleration:

$$v = v_0 + at \qquad x = x_0 + v_0 t + \frac{1}{2}at^2 \qquad v^2 - v_0^2 = 2a(x - x_0)$$

Curvilinear Motion of Particles

Rectangular coordinates

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k} = \dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k} = \ddot{x}\hat{\imath} + \ddot{y}\hat{\jmath} + \ddot{z}\hat{k}$$

Normal-tangential coordinates (planar)

$$\vec{v} = v\hat{e}_t$$

$$\vec{a} = a_t\hat{e}_t + a_n\hat{e}_n = \dot{v}\hat{e}_t + \frac{v^2}{\rho}\hat{e}_n$$

Polar coordinates

$$\vec{r} = r(t)\hat{e}_r$$

$$\vec{v} = \frac{d\vec{r}}{dt} = v_r\hat{e}_r + v_\theta\hat{e}_\theta = \dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta$$

$$\vec{a} = \frac{d\vec{v}}{dt} = a_r\hat{e}_r + a_\theta\hat{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

Relative Motion of Particles

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \qquad \vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad \vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$$

Kinetic Equations of Particles

$$\sum \vec{F} = m\vec{a}$$

Principle of Work and Energy

$$\sum T_1 + \sum U_{1-2} = \sum T_2 \qquad T = \frac{1}{2}mv^2$$

$$U_{1-2} = \int_{r_1}^{r_2} \vec{F}.d\vec{r} \qquad (U_{1-2})_{weight} = \pm W.\Delta y$$

$$(U_{1-2})_{Spring} = \frac{k}{2}({\Delta x_1}^2 - {\Delta x_2}^2) \qquad \sum T_1 + \sum V_1 = \sum T_2 + \sum V_2;$$

$$V_{weight} = \pm W.y \qquad V_{spring} = +\frac{k}{2}(\Delta x^2)$$

Linear Impulse and Momentum

$$\vec{L} = m\vec{v} \qquad I = \int_{t_1}^{t_2} \vec{F}\,dt \qquad \sum \vec{F} = \dot{\vec{L}} \qquad \sum m\vec{v}_1 + \sum \int_{t_1}^{t_2} \vec{F}\,dt = \sum m\vec{v}_2$$

Impact

Direct central impact

$$e = \frac{(v'_B) - (v'_A)}{(v_A) - (v_B)}$$

Oblique impact

$$e = \frac{(v'_B)_n - (v'_A)_n}{(v_A)_n - (v_B)_n}$$

Planar Kinematics of Rigid Bodies

Angular motion

$$\omega = \frac{d\theta}{dt} \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} \qquad \alpha d\theta = \omega d\omega$$

Angular motion with constant acceleration

$$\omega = \omega_0 + \alpha t \qquad \theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \qquad \omega^2 - {\omega_0}^2 = 2\alpha(\theta - \theta_0)$$

Relative motion analysis

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = \vec{v}_A + (\vec{\omega} \times \vec{r}_{B/A})$$

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} = \vec{a}_A + (\vec{a}_{B/A})_t + (\vec{a}_{B/A})_n = \vec{a}_A + \left(\vec{\alpha} \times \vec{r}_{B/A}\right) - \omega^2\left(\vec{r}_{B/A}\right)$$

Planar Kinetics of Rigid Bodies

Newton's second law for rigid body planar motion

$$I = I_G + md^2 \qquad I_G = m{k_G}^2$$

$$\sum \vec{F} = m\vec{a}_G \qquad \sum M_G = I_G\alpha \qquad \sum M_p = \sum (M_k)_p$$

Work and energy in rigid body planar motion

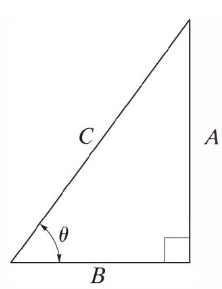
$$T = \frac{1}{2}m{v_G}^2 + \frac{1}{2}I_G\omega^2 \qquad U_{1-2} = \int_{r_1}^{r_2} \vec{F}.d\vec{r} \qquad U_{1-2} = \int_{\theta_1}^{\theta_2} M d\theta$$

Trigonometric Equations

$\sin\theta = \frac{A}{C}$, $\csc\theta = \frac{C}{A}$

$\cos\theta = \frac{B}{C}$, $\sec\theta = \frac{C}{B}$

$\tan\theta = \frac{A}{B}$, $\cot\theta = \frac{B}{A}$



$\sin^2\theta + \cos^2\theta = 1$

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

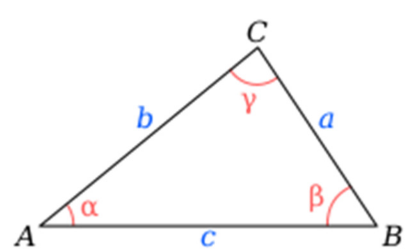
$\cos\theta = \pm\sqrt{\frac{1 + \cos 2\theta}{2}}$, $\sin\theta = \pm\sqrt{\frac{1 - \cos 2\theta}{2}}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$

Dynamics, Hibbeler, Appendix C

Law of Sines and Cosines



$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$