Problem ①

$$v\,dv = a_t\,ds$$

$$s = \frac{60}{360}(2\pi r) = \frac{1}{6}(2\pi \times 150) = 157.07\ m$$ 1 mark

$$\rightarrow \int_{40}^{V_B} v\,dv = \int_0^{157.07} -0.05\,s\,ds \rightarrow \frac{v^2}{2}\Big]_{40}^{V_B} = \frac{-0.05\,s^2}{2}\Big]_0^{157.07}$$ 2 marks

2 marks

$$\rightarrow \frac{V_B^2}{2} - \frac{40^2}{2} = \frac{-0.05}{2} \times 157.07^2 \rightarrow V_B = 19.4\ m/s$$ 1 mark

$$a_t\Big|_B = -0.05(157.07) = -7.85\ m/s^2$$ 1 mark

$$a_n\Big|_B = \frac{V_B^2}{\rho} = \frac{19.4^2}{150} = 2.44\ m/s^2$$ 2 marks

$$\vec{a}_B = a_t\,\hat{e}_t + a_n\,\hat{e}_n = -7.85\,\hat{e}_t + 2.44\,\hat{e}_n$$

$$|a_B| = \sqrt{(-7.85)^2 + (2.44)^2} = 8.22\ m/s^2$$ 1 mark

Problem ②

$\vec{V}_A = -125\cos 20\,\hat{i} - 125\sin 20\,\hat{j} = -117.46\,\hat{i} - 42.75\,\hat{j}$ m/s — 1 mark

$\vec{a}_A = -8\cos 20\,\hat{i} - 8\sin 20\,\hat{j} = -7.51\,\hat{i} - 2.73\,\hat{j}$ m/s² — 1 mark

$\vec{V}_B = 150\cos 45\,\hat{i} - 150\sin 45\,\hat{j} = 106.06\,\hat{i} - 106.06\,\hat{j}$ m/s — 1 mark

$\vec{a}_B = (a_B)_t\,\hat{e}_t + (a_B)_n\,\hat{e}_n = 0\,\hat{e}_t + \frac{150^2}{150}\,\hat{e}_n = 150\,\hat{e}_n$ m/s² — 1 mark

$\rightarrow \vec{a}_B = 150\cos 45\,\hat{i} + 150\sin 45\,\hat{j} = 106.06\,\hat{i} + 106.06\,\hat{j}$ m/s² — 2 marks

Now, relative motion analysis

$\vec{V}_{B/A} = \vec{V}_B - \vec{V}_A = (106.06\,\hat{i} - 106.06\,\hat{j}) - (-117.46\,\hat{i} - 42.75\,\hat{j})$ — 1 mark

$\rightarrow \vec{V}_{B/A} = 223.52\,\hat{i} - 63.31\,\hat{j}$ m/s

$\rightarrow |V_{B/A}| = 232.31$ m/s — 1 mark

$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A = (106.06\,\hat{i} + 106.06\,\hat{j}) - (-7.51\,\hat{i} - 2.73\,\hat{j})$

$\rightarrow \vec{a}_{B/A} = 113.57\,\hat{i} + 108.79\,\hat{j}$ — 1 mark

$\rightarrow |a_{B/A}| = 157.26$ m/s² — 1 mark

Problem (3)

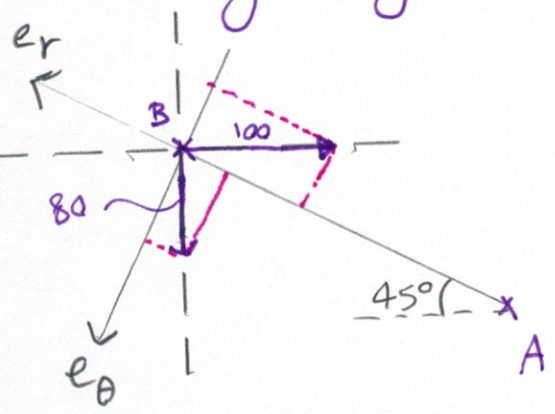
$\vec{V}_A = +80\,\hat{j}$ km/h (1 mark) $\quad \vec{a}_A = 0$ (1 mark)

$\vec{V}_B = +100\,\hat{i}$ km/h (1 mark) $\quad \vec{a}_B = -1000\,\hat{i}$ km/h^2 (1 mark)

$\vec{V}_{B/A} = \vec{V}_B - \vec{V}_a = 100\,\hat{i} - 80\,\hat{j}$ km/h (1 mark)

$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A = -1000\,\hat{i}$ km/h^2 (1 mark)

Velocity analysis



$(V_{B/A})_r = -100\cos 45 - 80\cos 45$

$\rightarrow (V_{B/A})_r = -127.27$ km/h (1 mark)

$(V_{B/A})_\theta = -100\sin 45 + 800\sin 45$

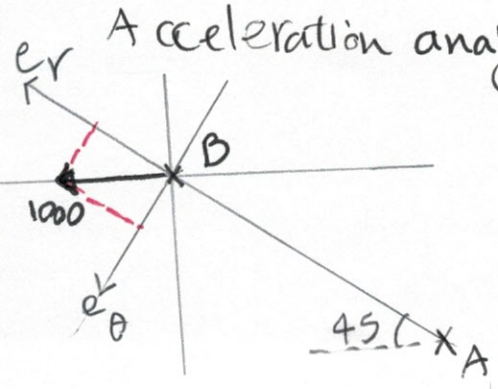
$\rightarrow (V_{B/A})_\theta = -14.14$ km/h (1 mark)

From equation sheet:

$V_r = \dot{r} \longrightarrow \dot{r} = -127.27$ km/h (1 mark)

$V_\theta = r\dot{\theta} \longrightarrow -14.14 = 2\dot{\theta} \longrightarrow \dot{\theta} = -7.07$ rad/h (1 mark)

Acceleration analysis



(1 mark)

$(a_{B/A})_r = 1000\cos 45 = 707.1$ km/h^2

$(a_{B/A})_\theta = 1000\sin 45 = 707.1$ km/h^2 (1 mark)

From equation sheet

$$(a_{B/A})_r = \ddot{r} - r\dot{\theta}^2 \rightarrow 707.1 = \ddot{r} - \left[2 \times (-7.07)^2\right]$$

$$\rightarrow \ddot{r} = 807.08 \text{ km/h}^2$$ 1.5 marks

$$(a_{B/A})_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \rightarrow 707.1 = 2\ddot{\theta} + \left[2 \times (-127.27) \times (-7.07)\right]$$

$$\rightarrow \ddot{\theta} = -546.24 \text{ rad/h}^2$$ 1.5 marks

Final numerical values in m,s units:

$\vec{V}_A = 22.22\hat{j}$ m/s $\quad \vec{a}_A = 0$

$\vec{V}_B = 27.78\,i$ m/s $\quad \vec{a}_B = -0.077\,\hat{i}$ m/s^2

$\vec{V}_{B/A} = 27.78\,\hat{i} - 22.22\,\hat{j}$ m/s

$\vec{a}_{B/A} = -0.077\,\hat{i}$

$(V_{B/A})_r = -35.35$ m/s and $\dot{r} = -35.35$ m/s

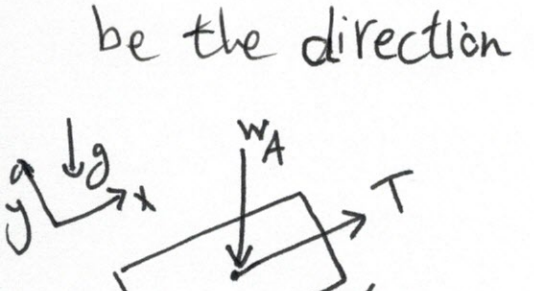
$(V_{B/A})_\theta = -3.93$ m/s and $\dot{\theta} = -0.00196$ rad/s

$(a_{B/A})_r = 0.0546$ m/s^2 and $\ddot{r} = 0.0823$ m/s^2

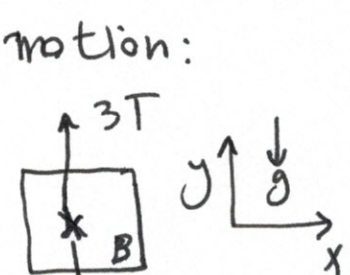
$(a_{B/A})_\theta = 0.0546$ m/s^2 and $\ddot{\theta} = -4.21 \times 10^{-5}$ rad/s^2

Problem ④

An impending motion analysis is required to determine if the system accelerates or not, and if so, what would be the direction of motion:



2 marks



2 marks

So, we assumed A is going down and B is going up.

for (A)

$$\nearrow \Sigma F_x = 0 \rightarrow T - W_A \sin 25 + F_A = 0 \quad (1)$$ 1 mark

$$+\uparrow \Sigma F_y = 0 \rightarrow N_A - W_A \cos 25 = 0 \rightarrow N_A = 15 \times 9.81 \times \cos 25 = 133.36\ N \quad (2)$$ 1 mark

for B

$$+\uparrow \Sigma F_y = 0 \rightarrow 3T - W_B = 0 \rightarrow T = \frac{8 \times 9.81}{3} = 26.16\ N \quad (3)$$ 1 mark

Sub $T = 26.16$ in (1) $\rightarrow 26.16 - (15 \times 9.81 \sin 25) + F_A = 0$

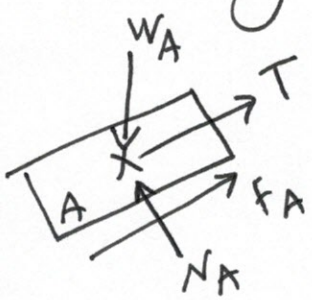
$\rightarrow F_A = 36.02\ N$ 1 mark

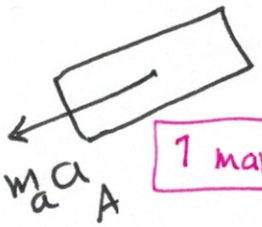
$F_{max} = \mu_s N = 0.2 \times 133.36 = 26.67\ N$

Since $F_A > \mu_s N_A \rightarrow$ the system moves. 2 marks

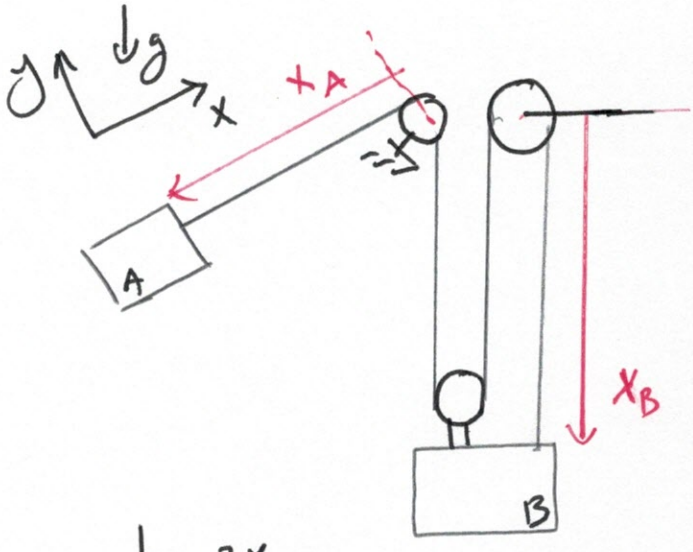
since the magnitud of F_A is pasitive $\rightarrow$ the assumed directions for motion are correct.

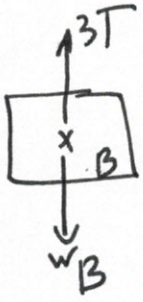
kinetic analysis

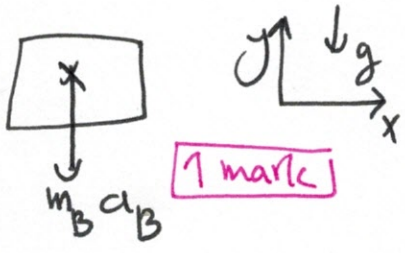




1 mark







1 mark

$L = 3x_B + x_A + cons$

$\rightarrow a_A + 3a_B = 0$ (4)

1 mark

(A)

$+\nearrow \Sigma F_x = m_A a_A \rightarrow T - W_A \sin 25 + F_A = -m_a a_A$

1 mark

$\rightarrow T - (15 \times 9.81 \times \sin 25) + (\mu_k N_A) = -15 a_A$ (5)

$+\uparrow \Sigma F_y = 0 \rightarrow N_A - W_A \cos 25 = 0$

$\rightarrow N_A = 133.36\ N$ (6)

1 mark

(B)

$+\uparrow \Sigma F_y = m_B a_B \rightarrow 3T - W_B = -m_B a_B$

$\rightarrow 3T - (8 \times 9.81) = -8 a_B$ (7)

1 mark

solving the system of eqs (4), (5), (6), (7)

$a_A = 1\ m/s^2 \downarrow$

1 mark

$a_B = -0.33\ m/s^2$

1 mark

$T = 27.05\ N$

1 mark