MATH 1402B **Term test #4** **19 March 2012**

Total marks: 28 marks

Non-programmable calculators are permitted. This test is closed book.

Supply your answers on this sheet, but TA's have extra paper if you need it.

PLEASE PRINT	First name	Last name	Student number

Please show your work where appropriate!

1. Consider the following system of 3 equations and 3 unknowns: $\begin{array}{r} 2x_1 + x_2 + 3x_3 = 0 \\ x_1 + x_2 + 2x_3 = -1 \\ x_2 - x_3 = -2 \end{array}$

   a. [2] Re-write the system in standard form.

   The system is already in standard form! ...

   b. [1] Write the augmented matrix for the system

   $$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 1 & 2 & -1 \\ 0 & 1 & -1 & 2 \end{array}\right]$$

   c. [4] Carry the augmented matrix to a R.E.F. (row-echelon form) **with 1's as leading entries**.

   The answer is not unique

   $$\left[\begin{array}{ccc|c} 2 & 1 & 3 & 0 \\ 1 & 1 & 2 & -1 \\ 0 & 1 & -1 & 2 \end{array}\right] \xrightarrow[R_1 \leftrightarrow R_2]{\text{SWAP}} \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 2 & 1 & 3 & 0 \\ 0 & 1 & -1 & 2 \end{array}\right] \xrightarrow{R_2' = R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & -1 & -1 & 2 \\ 0 & 1 & -1 & 2 \end{array}\right] \xrightarrow{R_2' = -R_2}$$

   $$\left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & 1 & -2 \\ 0 & 1 & -1 & 2 \end{array}\right] \xrightarrow{R_3' = R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & -2 & 4 \end{array}\right] \xrightarrow{R_3' = -\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & -1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 1 & -2 \end{array}\right]$$

2. [3] Carry this matrix to R.R.E.F. and solve the system. $\begin{bmatrix} 1 & -2 & 4 & 8 \\ 0 & 1 & 3 & 12 \\ 0 & 0 & 1 & -2 \end{bmatrix}$ $\xrightarrow[R_1' = R_1 - 4R_3]{R_2' = R_2 - 3R_3}$

   $$\begin{bmatrix} 1 & -2 & 0 & 16 \\ 0 & 1 & 0 & 18 \\ 0 & 0 & 1 & -2 \end{bmatrix} \xrightarrow{R_1' = R_1 + 2R_2} \begin{bmatrix} 1 & 0 & 0 & 52 \\ 0 & 1 & 0 & 18 \\ 0 & 0 & 1 & -2 \end{bmatrix} \Leftarrow$$

   This is unique

3. [6] Evaluate the following definite integrals:

a. $\int_0^{\ln 2} (e^x + 1)^2 e^x \, dx$	b. $\int_{-6}^{24} \sqrt{2x+16} \, dx$

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$u = e^x + 1 = g(x)$

$du = e^x \, dx$

$u_1 = g(0) = 2$

$u_2 = g(\ln 2) = e^{\ln 2} + 1 = 3$

$\Downarrow$

$$\int_2^3 u^2 \, du = \frac{1}{3}\left[u^3\right]_2^3$$

$$\ldots = \frac{1}{3}(27 - 8) = \boxed{\frac{19}{3}}$$

$$\int_{-6}^{24}\sqrt{2x+16}\,dx$$

$u = g(x) = 2x+16$
$du = 2\,dx \Rightarrow \frac{1}{2}du = dx$
$u_1 = g(-6) = 4$
$u_2 = g(24) = 64$

$$\Downarrow$$

$$\int_4^{64}\sqrt{u}\,du = \frac{2}{3}\left[u^{3/2}\right]_4^{64} = \frac{2}{3}(512-8) = \frac{1008}{3} = 336$$

4. [3] Determine *TC*, the Total Cost, if *MC*, the Marginal Cost, is given by: $MC = 7q^2 + 9q - 10$ and *TC*=5500 for *q*=12.

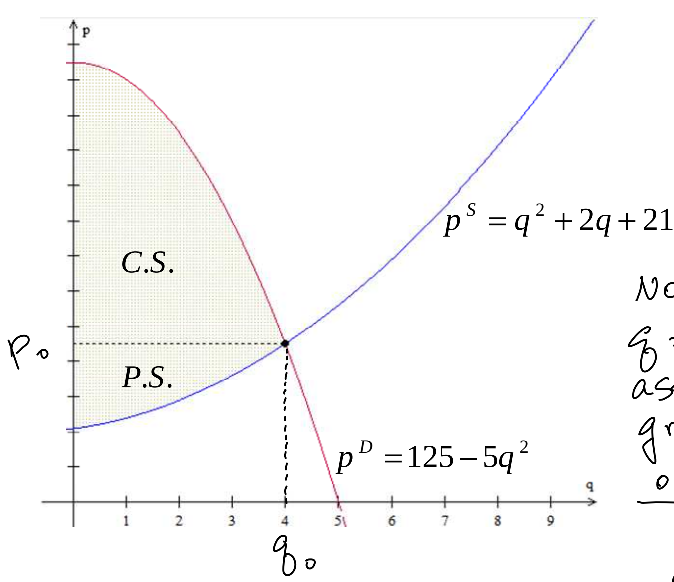
$$TC = \int MC\,dq = \int 7q^2+9q-10\,dq =$$

$$\ldots = \frac{7}{3}q^3 + \frac{9}{2}q^2 - 10q + C_1$$

but $TC\Big|_{q=12} = 5500 = \frac{7}{3}(12)^3 + \frac{9}{2}(12)^2 - 10(12) + C_1 \Rightarrow C_1 = 940$

$$\boxed{\therefore\ TC = \frac{7}{3}q^3 + \frac{9}{2}q^2 - 10q + 940}$$

5. [6] Determine the **<u>Producer's Surplus</u>** for the following case:



$(q_0, P_0) = (?,?) \Rightarrow$ set $p^S = p^D$

$\Rightarrow q^2 + 2q + 21 = 125 - 5q^2$

$6q^2 + 2q - 104 = 0$

$3q^2 + q - 52 = 0$

$q = \dfrac{-1 \pm \sqrt{1-(4)(3)(-52)}}{6} = \dfrac{-1 \pm 25}{6}$

$\therefore q = -\frac{13}{3}$ OR 4

NOTE: IF $q = 4$ is assumed from graph, it's ok also...

$\therefore P_0 = 125 - 5(4)^2 = 45$

$$\therefore P.S. = \int_0^4 45 - p^S\,dq =$$

$$\ldots = \int_0^4 45 - q^2 - 2q - 21\,dq = \int_0^4 24 - 2q - q^2\,dq =$$

$$\ldots = \left[24q - q^2 - \frac{1}{3}q^3\right]_0^4 = 24(4) - 4^2 - \frac{1}{3}4^3 = 96 - \frac{64}{3} =$$

$$= \frac{288-64}{3} = \frac{224}{3} = 74\tfrac{2}{3}$$

6. [3] Determine *TR*, the Total Revenue, if *MR*, the Marginal Revenue, is given by $MR = 12 - 4q$. Assume that *TR*=0 when *q*=0. Use the result obtained for *TR* to determine the inverse demand function (i.e. p^D=*p*).

$$TR = \int MR\,dq = \int 12 - 4q\,dq$$

$$= 12q - 2q^2 + C_1$$

But $TR\Big|_{q=0} = 0 = 12(0) - 2(0)^2 + C_1 \Rightarrow C_1 = 0$

$$\therefore \boxed{TR = 12q - 2q^2} = \underbrace{(12-2q)}_{p^D}q \Rightarrow \boxed{\therefore\ p^D = 12 - 2q}$$