MATH 1402B **Term test #3 – solution** **5 March 2012**

Total marks: 35 marks

Non-programmable calculators are permitted. This test is closed book.

<u>Supply your answers on this sheet</u>, but TA's have extra paper if you need it.

PLEASE PRINT	____________ First name	Last name	________ Student number

Please show your work where appropriate!

1. [10] Use *any method* to determine the K and L combination that ***maximizes profits***. Determine the resulting production output Q and profit $\prod$, given the following: $\boldsymbol{w} = 135$, $\boldsymbol{r} = 16$, $\boldsymbol{p} = 180$ and $Q = K^{2/5}L^{1/2} = K^{0.4}L^{0.5}$.

One method: Direct substitution:

$$\Pi = TR - TC = pQ - rK - wL = pK^{2/5}L^{1/2} - rK - wL$$

$$\Pi = 180K^{2/5}L^{1/2} - 16K - 135L$$

$$\frac{\partial \Pi}{\partial L} = 0 \Rightarrow 90K^{2/5}L^{-1/2} - 135 = 0 \Rightarrow 2K^{2/5}L^{-1/2} = 3 \quad (1)$$

$$\frac{\partial \Pi}{\partial K} = 0 \Rightarrow 72K^{-3/5}L^{1/2} - 16 = 0 \Rightarrow 9K^{-3/5}L^{1/2} = 2 \quad (2)$$

$$(1) \div (2) \quad \frac{2}{9}KL^{-1} = \frac{3}{2} \Rightarrow K = \frac{27}{4}L \quad (*)$$

substitute $\frac{27}{4}L$ for K in (1) (or (2)):

in (1): $2\left(\frac{27}{4}L\right)^{2/5}L^{-1/2} = 3 \Rightarrow 2\cdot\left(\frac{27}{4}\right)^{2/5}L^{-1/10} = 3$

$$\Rightarrow L^{-1/10} = \frac{3}{2}\cdot\left(\frac{4}{27}\right)^{2/5}$$

$$L = \left[\frac{3}{2}\cdot\left(\frac{4}{27}\right)^{2/5}\right]^{-10} = \boxed{36}$$

From (*) $\therefore K = \frac{27}{4}(36) = \boxed{243}$

Then : $Q = (243)^{2/5}\,36^{1/2} = 9 \cdot 6 = \boxed{54}$

and : $\Pi = pQ - wL - rK =$

$\ldots = 180(54) - 135(36) - 16(243)$

$\therefore \Pi = \boxed{972}$

2. [2] You are given the following information:
The wage rate is 128, while the rental rate is 32. Assume the standard, linear model for the total cost TC. You are required to maintain a production level of 256, given that $Q = 32K^{1/4}L^{3/4}$.

Write out the Lagrangian required to minimize *TC*. Do not solve the problem.

$TC = rK + wL = 32K + 128L$, and: $256 - 32K^{1/4}L^{3/4} = 0$

$$\therefore \mathcal{L} = 32K + 128L + \lambda\left(256 - 32K^{1/4}L^{3/4}\right)$$

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3. [6x2] Evaluate the antiderivative of the following:

a. $f(x) = -x^{3.5} - 3x^2 + 12x - 8 + \frac{12}{x^3} + 4e^x$ $F(x) = -\frac{x^{4.5}}{4.5} - x^3 + 6x^2 - 8x - \frac{6}{x^2} + 4e^x + C_1$	b. $f(x) = (2-4x)^2 = 4 - 16x + 16x^2$ $F(x) = 4x - 8x^2 + \frac{16}{3}x^3 + C_1$
c. $f(x) = \sqrt{\frac{2}{x^3}} = \sqrt{2}\, x^{-3/2}$ $F(x) = -2\sqrt{2}\, x^{-1/2} + C_1 = -\frac{2\sqrt{2}}{\sqrt{x}} + C_1$	d. $f(x) = \frac{\sqrt{x^3} - 5}{\sqrt{x^5}} = \frac{1}{x} - 5x^{-5/2}$ $F(x) = \ln\lvert x\rvert + \frac{10}{3} x^{-3/2} + C_1$ $F(x) = \ln\lvert x\rvert + \frac{10}{3\sqrt{x^3}} + C_1$
e. $f(x) = e^x + \pi^e - e^\pi$ $F(x) = e^x + \pi^e x - e^\pi x + C_1$	f. $f(x) = 5/x^3 = 5x^{-3}$ $F(x) = -\frac{5}{2}x^{-2} + C_1 = \frac{-5}{2x^2} + C_1$

4. [4] Determine the area under the curve $y = 1/x$ between the values $x = 1$ and $x = e^2$.

$$A = \int_1^{e^2} \frac{1}{x}\, dx = \Big[\ln\lvert x\rvert\Big]_1^{e^2} = \boxed{2}$$

5. Let $F(x) = \int \frac{3x^3 + 2x - 5}{x}\, dx$. $= \int 3x^2 + 2 - \frac{5}{x}\, dx$

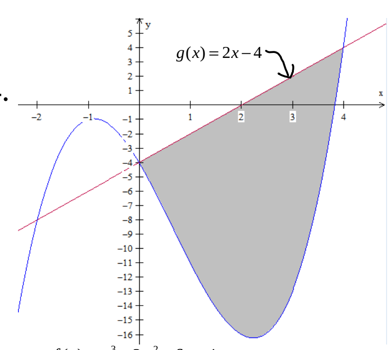
   a. [2] Determine the most general antiderivative $F(x)$.

   $$F(x) = x^3 + 2x - 5\ln\lvert x\rvert + C_1$$

   b. [2] Determine the constant of integration if $F(1) = -4$

   $$F(1) = -4 = 1^3 + 2(1) - 5\ln\lvert 1\rvert + C_1 \Rightarrow \boxed{C_1 = -7}$$

6. [3] Determine the area of the region that is enclosed between the curves $g(x) = 2x - 4$ and $f(x) = x^3 - 2x^2 - 6x - 4$ between x=0 and x=4.



$f(x) = x^3 - 2x^2 - 6x - 4$

$$f(x) = g(x) \Rightarrow x^3 - 2x^2 - 6x - 4 = 2x - 4$$
$$x^3 - 2x^2 - 8x = 0$$
$$x(x^2 - 2x - 8) = 0$$
$$x = 0 \ ; \ x = -2 \ ; \ x = 4$$

$$\therefore A = \int_0^4 (2x-4) - (x^3 - 2x^2 - 6x - 4)\, dx$$

$$= \int_0^4 8x + 2x^2 - x^3\, dx = \left[4x^2 + \frac{2}{3}x^3 - \frac{1}{4}x^4\right]_0^4 =$$

$$\ldots = 4(4)^2 + \frac{2}{3}4^3 - \frac{1}{4}4^4 = \boxed{42\tfrac{2}{3}}$$